

Methods for Solving Quadratic Equations

Strand:	Equations and Inequalities
Topic:	Solving quadratic equations over the set of complex numbers
Primary SOL:	All.3 The student will solve b) quadratic equations over the set of complex numbers.
Related SOL:	All.1b, c

Materials

- Different Methods of Solving Quadratics activity sheet (attached)
- Solving Quadratics activity sheet (attached)
- Creating Quadratic Equations activity sheet (attached)
- Graphing utility

Vocabulary

complex number, conjugate, discriminant, factor, imaginary number, quadratic equation, quadratic formula, quadratic function, solution, x-intercept, zero

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Discuss with students the various ways they have solved quadratic equations in the past. The methods should include factoring, using the quadratic formula, and completing the square when appropriate.
2. Display the examples found on the Different Methods of Solving Quadratics activity sheet. Model solving each problem, with students reproducing the solution on paper. In pairs or small groups, have students formulate a list of strategies (rules) for determining the most efficient method of solving quadratic equations. If students are having difficulty, have them to list pros and cons of each solving method. Teachers may split the activity sheet into stations for the different methods of solving.
3. Distribute copies of the Solving Quadratics activity sheet. Have students work in pairs to complete it, with Student A solving the odd-numbered problems and Student B solving the even-numbered problems. Then, have partners swap problems and check solutions by substituting the solutions back into the original equations. Circulate and support students as they work. Teachers should determine whether it is appropriate for students to check their solutions using a graphing utility.

4. Once students have developed some confidence, have them discuss their observations regarding calculating the discriminant to determine the number and types of solutions. Have them complete the table at right, in which the discriminant = D .

$D = b^2 - 4ac$	Nature of the solutions	Graphical Representation
$D > 0$		
$D = 0$		
$D < 0$		
$D =$ a perfect square		

Assessment

- **Questions**

- How are the solutions of a quadratic function and the discriminant of the corresponding quadratic equation related? Explain your reasoning.
- For quadratic equations in the form $ax^2 + bx + c = 0$, what effect do changes in each of the constants have on the number and nature of the solutions?
- How can you determine whether a quadratic equation can be factored using only the discriminant?

- **Journal/writing prompts**

- A classmate does not know how to solve a quadratic equation, and you must teach him/her. Write the steps you will tell him/her to use.
- List the various ways to solve a quadratic equation. Then, describe the one you think is best, and justify why.

- **Other Assessments**

- Once students have had experience solving quadratics with complex solutions, ask them to make observations. They should observe that if a complex number in the form $a + bi$ is a solution, then $a - bi$ is also a solution—that is, such complex number solutions appear in conjugate pairs. Have students explain why.

Extensions and Connections

- Have students complete the Creating Quadratic Equations activity sheet (or another set of solutions to quadratic equations) and create equations from the solutions.
- Derive the quadratic formula using completing the square.
- In physics, quadratic equations are used to model the position of a ball or other projectile thrown in the air. If $y = ax^2 + bx + c$ models the position of a ball, y represents the height above the ground, and x represents the time in seconds, what must we know about the discriminant?

Strategies for Differentiation

- Reduce the number of questions on the Solving Quadratics activity sheet or provide an answer bank.

Mathematics Instructional Plan – Algebra II

- Before students solve quadratic equations, have them identify a , b , and c , and substitute these constants into a part of the quadratic formula. Then, have the students substitute them into the entire formula.
- Have students sing a song to help them remember the quadratic formula.
- Review simplifying radicals, as needed.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Different Methods of Solving Quadratics

1. Use factoring to solve $2x^2 - x - 1 = 0$.
2. Use the quadratic formula to solve $2x^2 - x - 1 = 0$.
3. Which method was easier to solve $2x^2 - x - 1 = 0$? Why?
4. Using the function $y = x^2 - 1$, discuss ways to solve $x^2 - 1 = 0$ and then solve. Check your solutions with a graphing utility.
5. Using the function $y = x^2 + 1$, discuss ways to solve $x^2 + 1 = 0$. Use the quadratic formula to solve.
6. Using the function $y = x^2 + 2x - 1$, discuss ways to solve $x^2 + 2x - 1 = 0$. Use completing the square to solve.
7. Use the quadratic formula to solve $x^2 + x + 1 = 0$.

8. Determine the discriminant of $3x^2 + 5x + 2 = 0$. Next, determine the best method of solving and then solve. Check your solutions with a graphing utility.

9. Determine the discriminant of $3x^2 + x + 2 = 0$. Next, determine the best method of solving and then solve. Check your solutions with a graphing utility.

Solving Quadratics

For each of the following quadratics, first determine the discriminant, and then determine the best method of solving and solve. Show all steps.

1. $2x^2 + 4x + 15 = 0$

2. $5x^2 = 2x - 8$

3. $6x^2 - x + 24 = 0$

4. $15x^2 + 2x + 1 = 0$

5. $9x^2 + 3x + 4 = 0$

6. $3x^2 - 2x + 4 = 0$

7. $3x^2 - 2x + 1 = 0$

8. $2x^2 + 3x = -8$

9. $3x^2 + 4x = -2$

10. $2x^2 - 3x + 5 = 0$

11. $3x(x + 1) = x - 5$

12. $2x^2 + 8 = x$

13. $7x - 13 = x^2$

14. $x^2 + 3x + 5 = 0$

15. $x^2 + 4 = 2x$

Creating Quadratic Equations

Construct a quadratic equation for each of the following solutions.

1. -2 and 5

2. $\sqrt{5}$ and $-\sqrt{5}$

3. $\frac{-1+\sqrt{2}}{3}$ and $\frac{-1-\sqrt{2}}{3}$

4. $3i$ and $-3i$

5. $2-5i$ and $2+5i$

6. $\frac{1}{2} + \frac{i\sqrt{3}}{2}$ and $\frac{1}{2} - \frac{i\sqrt{3}}{2}$