Radical Equations

Strand: Equations and Inequalities

Topic: Solving equations containing radical expressions.

Primary SOL: AII.3 The student will solve

d) The student will solve equations containing radical expressions.

Related SOL: AII.1b

Materials
- Solving Radical Equations (Introductory Exercise) activity sheet (attached)
- Steps for Solving Radical Equations Algebraically activity sheet (attached)
- Solving Radical Equations: Practice Problems with Hints activity sheet (attached)
- Graphing utility

Vocabulary
cube, cube root, exponent, extraneous solution, index, power, radical, radical algebraic equations, radicand, square, square root

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Distribute the Solving Radical Equations (Introductory Exercise) activity sheet. Have students complete the activity, working individually and then in pairs to share and confirm or revise their responses. Have them place emphasis on the justification of their answers. Follow with a class discussion of each problem. Pay attention to question 5, in which an untrue statement leads to an extraneous solution. Review the meaning of the term *extraneous solution.*

2. To explore the algebraic and graphical methods for solving rational expressions, begin with the algebraic. Distribute the Steps for Solving Radical Equations Algebraically activity sheet. As you lead students through the examples, encourage students to work with their partners to monitor and communicate what is happening. After each example, have a student pair come up and work the similar, accompanying problem. The variety of problems is meant to encompass the scope of typical Algebra II problems.

3. Use some of the problems from the Steps for Solving Radical Equations Algebraically activity sheet to introduce students to solving radical equations by graphing. Have students graph $f(x)=\sqrt{3x-5}$. Ask students about the domain of the function. Have the students evaluate $f(3)$. Ask students to find the value of x when $f(x)=2$. Discuss how these two are related on the graph. Ask the students to find the solution to the equation $\sqrt{3x-5}=2$ using the graph and to describe how they would use the graph to solve the equation. Have students use a graph to find the solution to $\sqrt{x+2}=x$. Discuss how this problem is similar to and different from the first example. Remind the students of the importance of identifying the domain for each function and how that relates to the possible solutions to the equation and any extraneous solutions.

4. Distribute the Solving Radical Equations: Practice Problems with Hints activity sheet, and have students complete it. (Note: This set of problems involving radical equations contains some that are a bit more challenging.)

Assessment

- **Questions**
  - How can you determine the solution set to a radical equation algebraically?
  - How can you determine the solution set to a radical equation graphically?
  - What would you do to eliminate the radicals in an equation such as $\sqrt[3]{x-1} = \sqrt{8}$?

- **Journal/writing prompts**
  - Explain how you can use a graph's points of intersection to solve a radical equation.
  - In your own words, explain what is meant by the term *extraneous solution.* Is it a solution or not? Explain why.

- **Other Assessments**
  - Give students solution sets, and ask them to create matching radical equations. (Note: Such open-ended problems allow students to be creative and differentiate the task based upon their own level of understanding.)

Extensions and Connections

- Give advanced students equations with multiple radicals.
- Guide students to make connections to graphing functions containing radicals, paying particular attention to restrictions on the domain.

Strategies for Differentiation

- Construct additional introductory problems to reinforce similar concepts.
- Create an additional handout similar to the Steps for Solving Radical Equations Algebraically activity sheet with examples in the left-hand column and similar radical equations in the right-hand column for students to solve.
- Have students create flash cards, each with a radical equation on one side and the first step on the other, to help them take the initial steps.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Virginia Department of Education © 2018

Solving Radical Equations
(Introductory Exercise)

Determine which of the following is true and justify your answers.

1. $x = 2$ is a solution to $\sqrt{3x - 2} = -2$

2. $x = 2$ is a solution to $\sqrt{3x - 2} = 2$

3. $x = 4$ is a solution to $\sqrt[3]{5x^2 - 4x} = x$

4. $a = b \Rightarrow a^n = b^n$

5. $a^n = b^n \Rightarrow a = b$

6. $x^{\frac{1}{2}} = \sqrt{x}$

7. $3\sqrt{2x} + \sqrt[3]{x} = 14$ when $x = 8$

8. $x - \sqrt{x} - 42 = 0$ when 36 or 49

9. $\sqrt[3]{x^2} = \sqrt{x^3}$ for infinitely many values of *x*

10. $\sqrt{x^2} = |x|$

Steps for Solving
Radical Equations Algebraically

Example 1: $\sqrt{3x-5}=5$

Step 1: Square both sides to eliminate the radical.
$$\left(\sqrt{3x-5}\right)^2=(5)^2$$

Step 2: Simplify and solve the familiar equation.
$$\Rightarrow 3x-5=25$$
$$\Rightarrow 3x=30$$
$$\Rightarrow x=10$$

Step 3: Verify the solution.
Does $\sqrt{3(10)-5}=5$?

Problem 1: Now, you follow the steps to solve $\sqrt{5x+1}=4$.

Step 1:

Step 2:

Step 3:

Example 2: $\sqrt[3]{5x+2}=3$

Step 1: Cube both sides to eliminate the radical.
$$\left(\sqrt[3]{5x+2}\right)^3=3^3$$

Step 2: Simplify and solve the familiar equation.
$$\Rightarrow 5x+2=27$$
$$\Rightarrow 5x=25$$
$$\Rightarrow x=5$$

Step 3: Verify the solution.
Does $\sqrt[3]{5(5)+2}=3$?

Problem 2: Now, you follow the steps to solve $\sqrt[3]{2x-1}=5$.

Step 1:

Step 2:

Step 3:

Example 3: $\sqrt{x+2}=x$

Step 1: Square both sides to eliminate the radical.
$$\left(\sqrt{x+2}\right)^2=x^2$$

Step 2: Simplify and solve the familiar equation.
$$\Rightarrow x+2=x^2$$
$$\Rightarrow 0=x^2-x-2=(x-2)(x+1)$$
$$\Rightarrow x=2 \ or \ x=-1$$

Step 3: Verify the solution.
Check with $x=2$ $\sqrt{2+2}=2$: True
Check with $x=-1$ $\sqrt{-1+2}=-1$: False
So, the only solution is $x=2$.
(−1 is extraneous.)

Problem 3: Now, you follow the steps to solve $\sqrt{2x+8}=x$.

Step 1:

Step 2:

Step 3:

Solving Radical Equations:
Practice Problems with Hints

Solve each of the following and check your solutions.

1. $2\sqrt{x} - 5 = 3$ (Hint: First solve for the $\sqrt{x}$.)

2. $x - 2\sqrt{x} - 15 = 0$ (Hint: Let $\sqrt{x} = n \implies x = n^2$, and then substitute.)

3. $\sqrt{x^2 + 2} = x + 1$ (Hint: Don't forget, when you square a binomial, it becomes a trinomial.)

4. $\sqrt{2x+1} = 2x - 1$ (Hint: Make sure you check for extraneous solutions.)

5. $x^{\frac{1}{2}} = x - 2$ (Hint: Remember what $x^{\frac{1}{2}}$ equals.)

Challenge Problems

6. The distance from a point on the curve $y = \sqrt{x}$ to the point (2,0) is equal to 2 at what points on the curve?
   (Hint: Draw a picture.)

7. Finish the right side of the equation $\sqrt{3x-5} = a$ by finding a so that $x = 7$ is a solution to the equation.
   (Hint: If $x = 5$ satisfies the equation, what does that mean?)

8. Solve $\sqrt{x+1} = 4 - 2x$
   (Hint: Don't forget, when you square a binomial, it becomes a trinomial.)

9. Solve $\sqrt{2x-5} = 3 - x$, and verify your solution, using a graphing utility.
   (Hint: You will need to use the quadratic formula.)

10. Solve $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 2 = 0$. (Hint: Let $u = x^{\frac{1}{3}}$, and solve for u first.)