Nonlinear Systems of Equations

Strand: Equations and Inequalities

Topic: Solving nonlinear systems of equations

Primary SOL: AII.4 The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.

Related SOL: AII.3

Materials

- Points of Intersection activity sheet (attached)
- Linear-quadratic System Practice activity sheet (attached)
- Quadratic-quadratic System Exploration activity sheet (attached)
- Quadratic-quadratic System Practice activity sheet (attached)
- Graphing utility

Vocabulary

addition method, coordinates, inverse system, linear, linear-quadratic system, linear system, nonlinear system, ordered pair, parabola, points of intersection, quadratic, quadratic system, quadratic-quadratic system, quadratic term, substitution method

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Time: 90 minutes

1. Review linear systems of equations and the three approaches students used in Algebra I to solve them (i.e., substitution, addition method, and graphing). Show an example of each method.
2. Distribute the Points of Intersection activity sheet. Have students work in pairs to determine the possible number of points of intersection (the number of solutions to a corresponding system) for each graph. Students will write their solutions and justifications for each example.
3. Model the algebraic and graphing methods for solving a linear-quadratic system using a graphing utility. Distribute the Linear-quadratic System Practice activity sheet. Have students work in pairs (Partner A and Partner B) to solve each problem. Instruct Partner A to solve the problem algebraically and Partner B to solve the same problem graphically. Then, have partners confer and confirm the solution. Circulate and give help as needed.
4. Distribute the Quadratic-quadratic System Exploration activity sheet, and have students work in small groups to complete it. When they have finished, discuss the results as a whole class. Then, model algebraic methods for solving quadratic-quadratic systems.
5. Distribute the Quadratic-quadratic System Practice activity sheet. Have students work in pairs to solve each problem. Direct Partner A to solve the problem algebraically and Partner B to solve the same problem graphically. Then, have partners confer and confirm the solution. Circulate and give help as needed.

Assessment

- **Questions**
  - What allows us to use substitution to solve a linear system of equations? What allows us to add two equations together to solve a system?
  - How do you decide which method is best for each type of problem?
- **Journal/writing prompts**
  - Write the steps you would instruct someone to use to solve a linear-quadratic system.
  - Explain how you know that you have no solutions when solving a quadratic-quadratic system algebraically.
  - Explain how you know that you have no solutions when solving a quadratic-quadratic system graphically.
- **Other Assessments**
  - Provide students with ordered-pair solutions to nonlinear systems of equations. Have them create the systems of equations to correspond to the given solutions.

Extensions and Connections

- Have students find a real-world problem that can be solved with a nonlinear system of equations.

Strategies for Differentiation

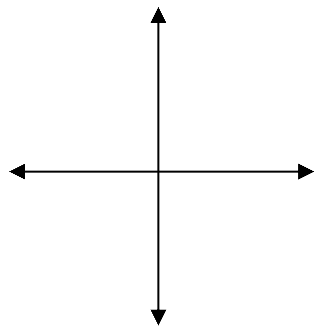
- Use a conic section graphing application on a graphing utility to help students visualize solutions to nonlinear systems of equations.
- Create a matching game to match graphs with nonlinear systems of equations.
- Create a sentence frame to accompany the Points of Intersection activity sheet. The sentence frame might be, "My <u>parabola</u> and my <u>circle</u> intersect at <u>two</u> point(s)."

Note: The following pages are intended for classroom use for students as a visual aid to learning.

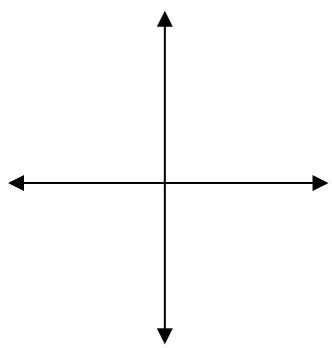
Points of Intersection

Explore the possible number of points of intersection of two lines.

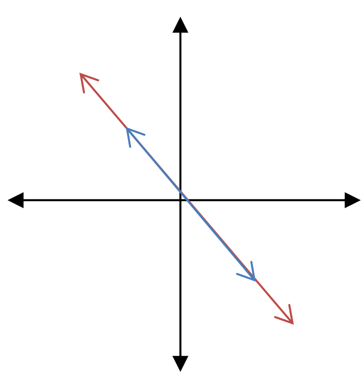
1. Draw two parallel lines on the coordinate grid below. How many points of intersection exist? Explain how you arrived at your solution.



2. Draw two intersecting lines on the grid below. How many points of intersection exist? Explain how you arrived at your solution.

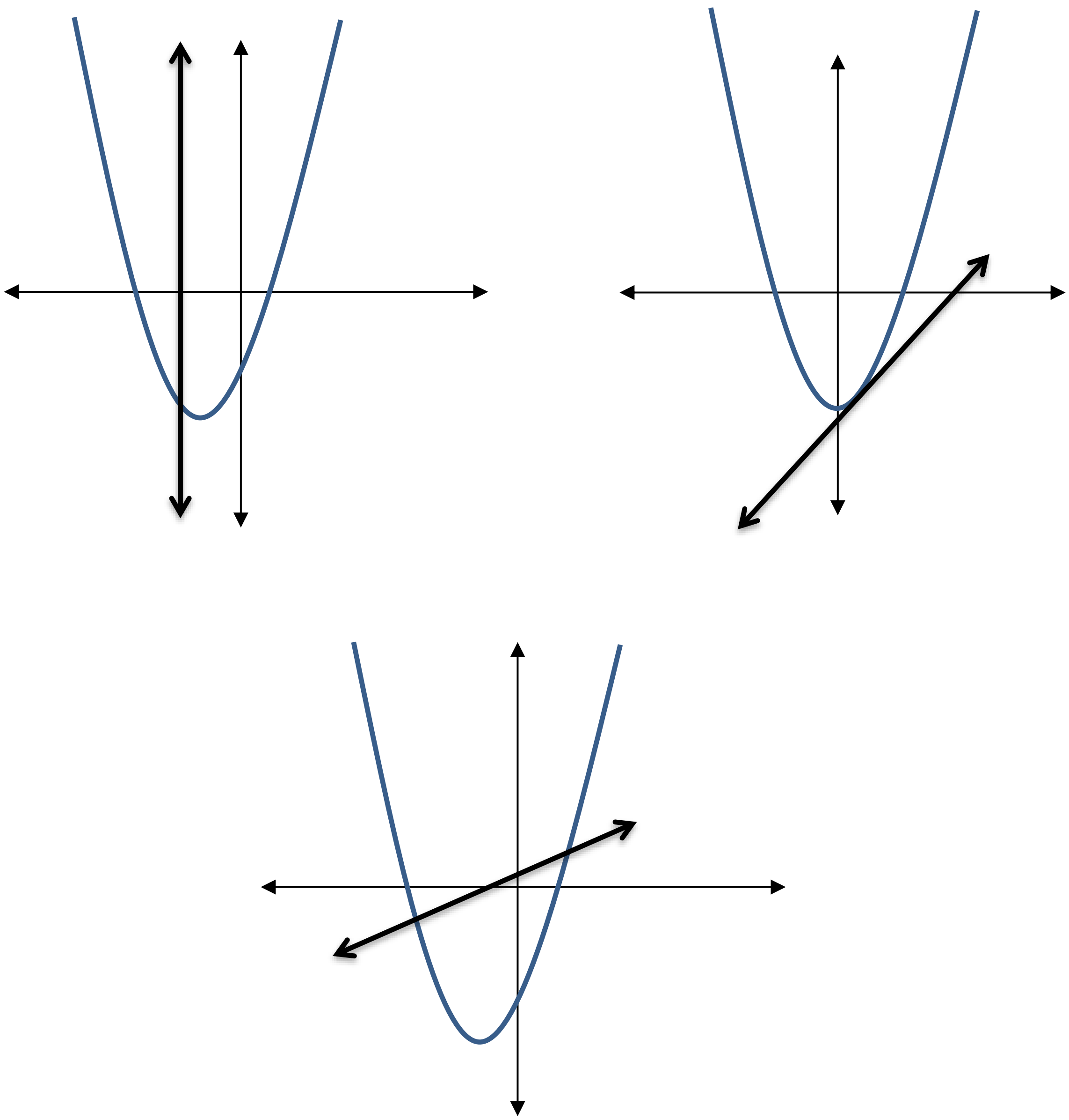


3. On the graph below, there are two overlapping lines. How many points of intersection exist? Explain how you arrived at your solution.



Explore the possible points of intersection of a line and a parabola.

4. The graphs below contain a parabola and a line. How many points of intersection are there for each of the graphs? Explain how you arrived at your solution.



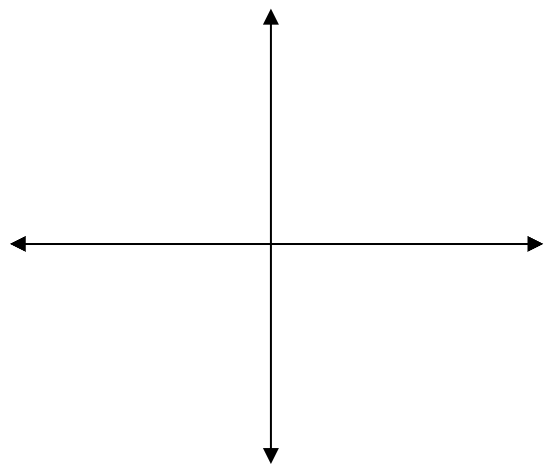
Linear-quadratic System Practice

1. $\begin{cases} y = x^2 + 5x - 10 \\ y = 2x + 18 \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

Show the solution on the graph below.

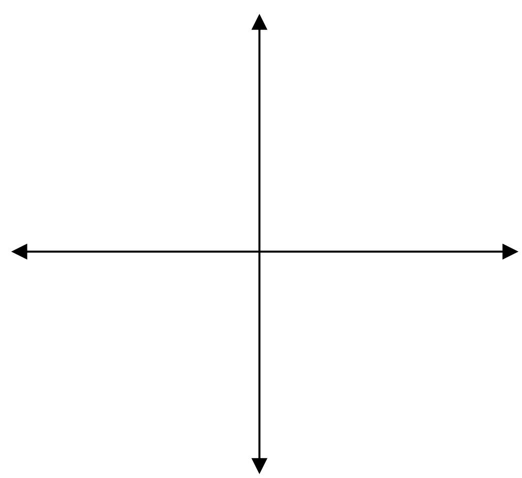


2. $\begin{cases} y = x^2 - 10x + 17 \\ 3x + y = 11 \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

Show the solution on the graph below.

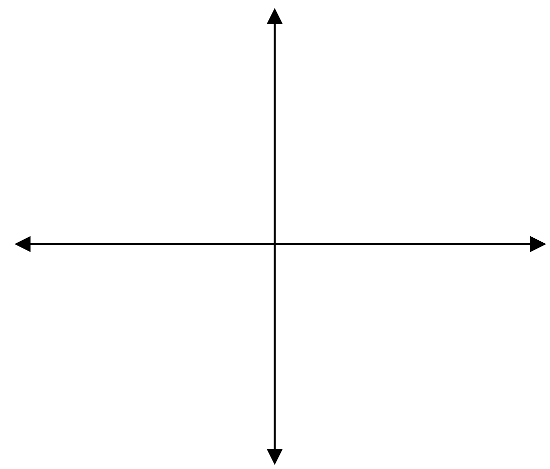


3. $\begin{cases} y = \frac{1}{2}(x-3)^2 + 4 \\ x + 2y = 13 \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

Show the solution on the graph below.

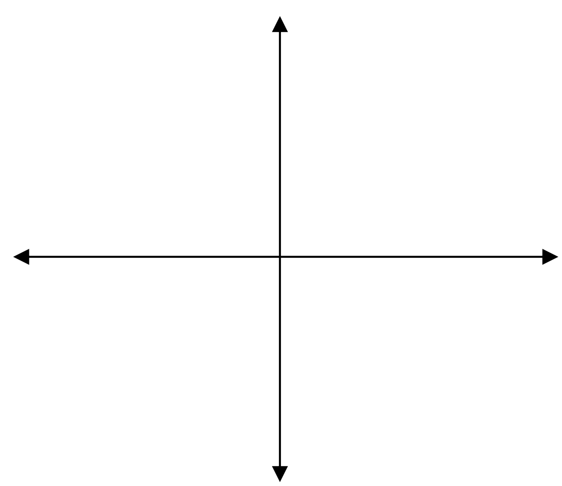


4. $\begin{cases} \text{y} = \text{x}^2 + 4\text{x} - 2 \\ \text{y} = 6\text{x} - 3 \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

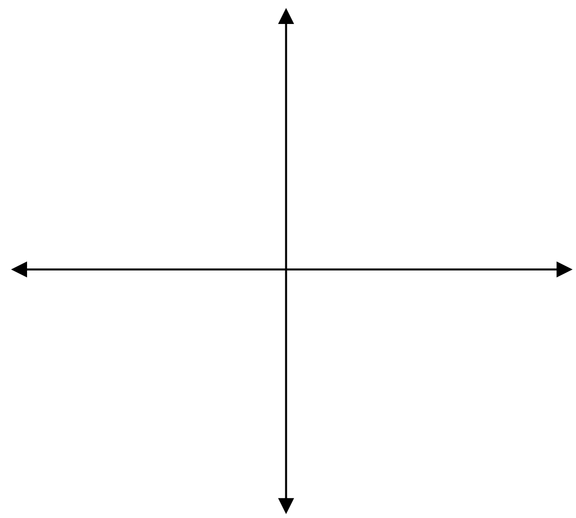
Show the solution on the graph below.



Quadratic-quadratic System Exploration

1. Given the system $\begin{cases} y=(x-3)^2+4 \\ y=-(x-1)^2+8 \end{cases}$, complete the following:

   a) Sketch both equations on the coordinate system shown at right.



   b) Describe graphically and algebraically what is meant by a solution to a system of equations.

   c) How many solutions does this system have?

   d) Solve this system algebraically, using the process of substitution.

   e) Graph this system using a graphing utility. Describe the procedure you used.

2. If possible, rewrite the system shown above, keeping one of the equations the same but changing the other equation to satisfy each of the following stated requirements. If the change is not possible, explain why.
   a) There are exactly three solutions for the system.

   b) There are exactly two solutions for the system.

   c) There is only one solution for the system.

   d) There is no solution for the system.

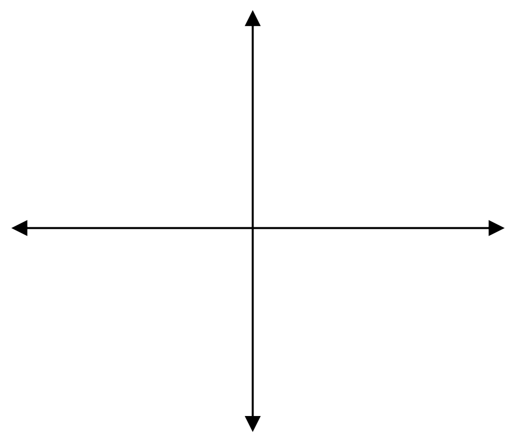
Quadratic-Quadratic System Practice

1. $\begin{cases} y = x^2 - 2x + 4 \\ y = -x^2 - 2x + 4 \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

Show the solution on the graph below.

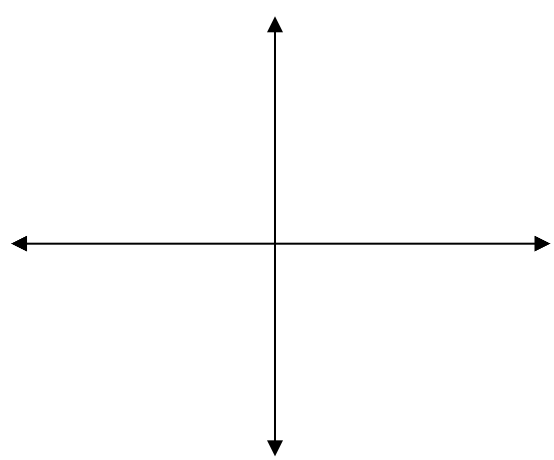


2. $\begin{cases} y = x^2 - 4x + 5 \\ y = -x^2 + 5 \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

Show the solution on the graph below.

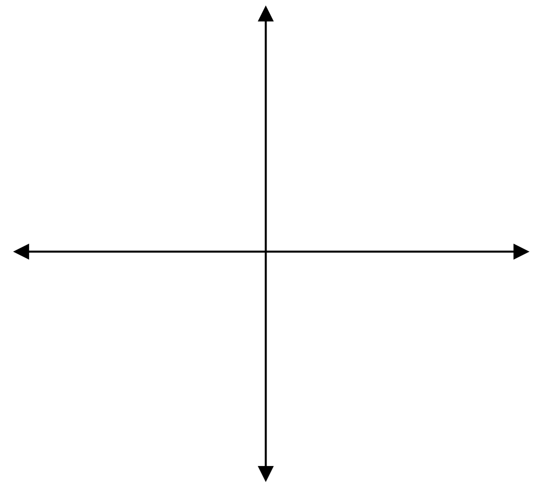


3. $\begin{cases} \mathrm{y} = -\mathrm{x}^2 + 9 \\ \mathrm{y} = \mathrm{x}^2 + 1 \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

Show the solution on the graph below.



4. $\begin{cases} y = 3x^2 - x - 2 \\ x^2 = y - 2x \end{cases}$

Solve algebraically by substitution.

Solve by using a graphing utility.

Show the solution on the graph below.