

# Functions: Domain, Range, Continuity and End Behavior

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<b>Strand:</b>	Functions
<b>Topic:</b>	Finding domain and range, continuity, and end behavior for a given function
<b>Primary SOL:</b>	All.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include a) domain, range, and continuity; and h) end behavior.
<b>Related SOL:</b>	All.6

## Materials

- Functions Characteristics Examples activity sheet (attached)
- Where to Begin and End activity sheet (attached)
- Domain and Range activity sheet (attached)
- Graphing utility
- Demonstration tool (e.g., document camera, graphing software)

## Vocabulary

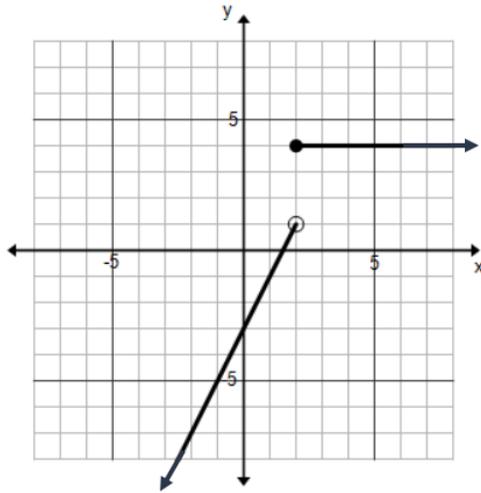
*continuous, dependent variable, discontinuous, domain, end behavior, horizontal asymptote, independent variable, input, interval notation, negative infinity, nonremovable discontinuity, output, positive infinity, range, removable discontinuity, vertical asymptote*

## Student/Teacher Actions: What should students be doing? What should teachers be doing?

*Time: 90 minutes*

1. The first activity for this lesson involves the teacher demonstrating how to graph certain functions and discussing how to find the domain, range, continuity, and end behavior. Students can use the Function Characteristics Examples activity sheet to record their notes during the class discussion if needed.
2. Have students graph the first example function,  $f(x) = x^2$ , while you demonstrate the graphing steps with a graphing utility. Students will benefit from seeing graphs displayed with different graphing utilities. Have students sketch their graph on the Function Characteristics Examples activity sheet. Then, have students discuss with partners the definitions of domain and range and determine the domain and range of the quadratic function in example 1.
3. Discuss the difference between a continuous function and a discontinuous function. Have students determine whether the quadratic function in example 1 is a continuous function.
4. Demonstrate, and have students copy into notes, how to express the domain  $\{x \mid x \in \mathbb{R}\}$  and range  $\{f(x) \mid f(x) \geq 0\}$  in both set notation and interval notation. In interval notation, the domain is  $(-\infty, \infty)$  and the range is  $[0, \infty)$ .

5. Discuss the end behavior of the function, both as  $x$  approaches negative infinity and as it approaches positive infinity. Demonstrate the proper notation for expressing end behavior. ( $x \rightarrow \infty, f(x) \rightarrow \infty$  and  $x \rightarrow -\infty, f(x) \rightarrow \infty$ )
6. Have students use a graphing utility to graph example 2,  $f(x) = \sqrt{2x-6}$ , as you graph it. Use a demonstration tool to display the graph to the class. Ask, “Why do we see the graph in the first quadrant only? How would the domain and range be different from the parabola?” Use think-pair-share to allow students to think about their answer, share it with a partner, and then share with the class. Have students sketch the graph and write the domain and range in both set and interval notation.
7. Ask students about the continuity of the function. You should note for students that the function is continuous over its domain. Ask, “What about the end behavior? What happens to the function as ‘ $x$ ’ approaches infinity? What about as ‘ $x$ ’ approaches negative infinity?” Clarify for students that the function does not have end behavior as  $x$  approaches negative infinity because that is not part of its domain.
8. Have students use a graphing utility to graph example 3,  $f(x) = \frac{x}{x+2}$ , as you graph it. Use a demonstration tool to display the graph to the class. Ask students, “What do you notice about this function?” Students should mention that the graph seems to be in pieces. Ask, “What does this mean about the continuity of the function?” Students should recognize that the function is discontinuous. Tell students that this is an example of nonremovable discontinuity.
9. Ask, “Where is there a gap in the pieces of the function?” Students should recognize that there is a gap in the function at  $x = -2$ . Ask students to think about why there might be a gap there. “What about the function might cause it to have a gap at  $x = -2$ ?” Students should notice that  $x = -2$  would make the denominator zero and would cause the function to be undefined there. Ask, “How would this affect the domain of this function?” Identify this as the vertical asymptote of the function. Model how to write the domain of the function for students in both interval and set notation.
10. Ask students whether they see any other possible asymptote lines. Students should respond with the horizontal asymptote line at  $y = 1$ . Identify that as the horizontal asymptote. Ask, “Why is there a gap at  $y = 1$ ? Does this change the domain or the range?” Have students write the range and check it with a partner. Discuss why there might be a gap at  $y = 1$ . Set the function equal to 1 and have students try to solve the equation. (While asymptote lines may be covered before or after this lesson, this is a good opportunity to introduce or review that concept, particularly for rational functions.)
11. Ask students to identify the end behavior of this function for example 3. Remind students that the end behavior is what happens as  $x$  approaches positive or negative infinity and not what happens in the middle of the graph. Ask, “What do you notice about the end behavior and the horizontal asymptote?”
12. Show students a graph of the function  $f(x) = \begin{cases} 2x-3, & x < 2 \\ -4, & x \geq 2 \end{cases}$  as shown below and in example 4.



Ask students to write the domain and range, end behavior, and describe whether it is continuous. Students should notice that the domain is all real numbers, but the range is actually over two different intervals. The range in set notation is  $\{y \mid y < 1 \text{ or } y = 4\}$  and in interval notation  $(-\infty, 1) \cup [4]$ . Point out to students that one of the endpoints is open while the other is closed. Students should recognize that this function is discontinuous. Define this as jump (not removable) discontinuity.

13. Distribute the Where to Begin and End activity sheet and have students work in pairs or groups to complete it.
14. Distribute the Domain and Range activity sheet for review the next day.

### Assessment

- **Questions**

- What are the domain and range for  $f(x) = \sqrt{x-4}$  ?
- How would you describe the end behavior of  $f(x) = 2^x$  ?
- Draw a function that has a nonremovable discontinuity at  $x = 3$  .

- **Journal/writing prompts**

- Create and describe a function, both algebraically and graphically, that would have end behavior of  $x \rightarrow \infty, f(x) \rightarrow -\infty$  and  $x \rightarrow -\infty, f(x) \rightarrow -\infty$ , domain of  $(-\infty, \infty)$  and range of  $(-\infty, 5]$  .
- Discuss whether it is possible for a function to have a removable discontinuity at  $x = -3$  and still have a domain of all real numbers. Justify your answer in at least two ways.
- Compare the end behaviors of a quadratic function and its reflection over the  $x$ -axis.

- **Other Assessments**

- Draw a function with a domain of  $[-4, 1]$  and a range of  $[2, 6]$  .

### Extensions and Connections

- Have students state the domain and range for a circle with center (2, 5) and radius 4.
- Present the following problem and discuss the domain and range of the function for this situation.

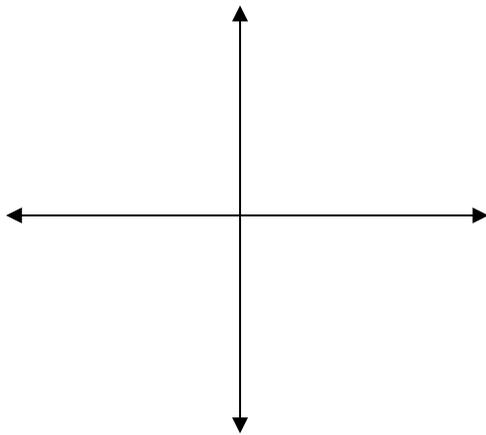
The school is planning a trip to visit a college about three hours away. They are renting a coach bus that holds 40 students. The administration has determined that it will cost \$12.50 per person to rent the bus and a flat fee of \$25 to park. The function that models this is  $c(s) = 12.50s + 25$ , where  $s$  represents the number of students on the trip and  $c(s)$  represent the cost. What is the domain and range of the function for this situation?

### **Strategies for Differentiation**

- Have students model with their arms the end behavior of a polynomial function, depending on the sign of the leading coefficient and the degree of the polynomial.
- Have students model a given end behavior using their arms.
- Have students cover up the area of the graph not included in the domain or range to help them identify the regions.
- Introduce domain and range by having students describe graphs of functions to each other using the appropriate vocabulary, similar in format to the “Pyramid” game show, or Pictionary with graphs by having students describe functions to each other using the appropriate vocabulary.

## Function Characteristics Examples

Example 1



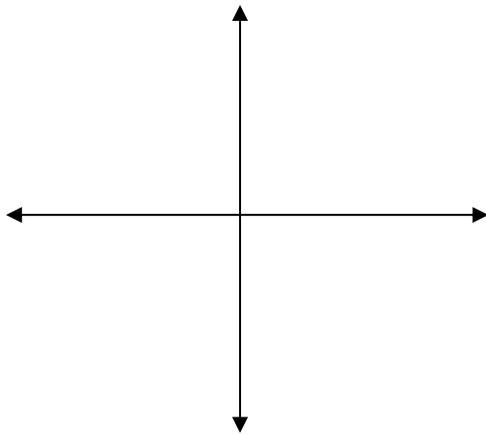
Domain:

Range:

Is the function continuous?

End Behavior

Example 2



Domain:

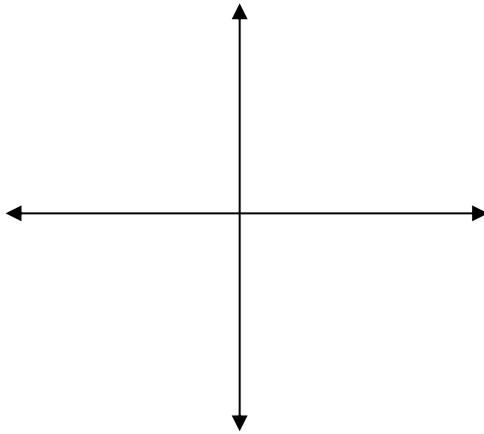
Range:

Is the function continuous?

End Behavior

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Example 3



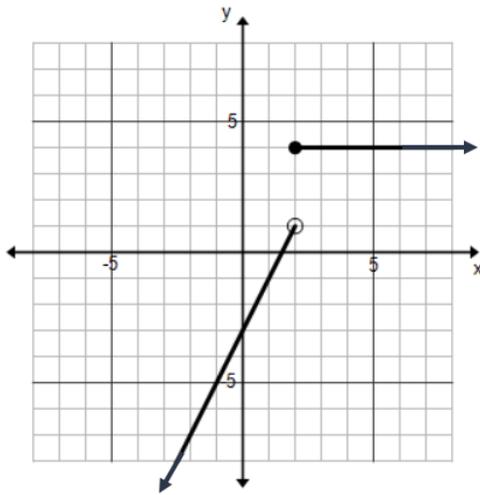
Domain:

Range:

Is the function continuous?

End Behavior

Example 4



Domain:

Range:

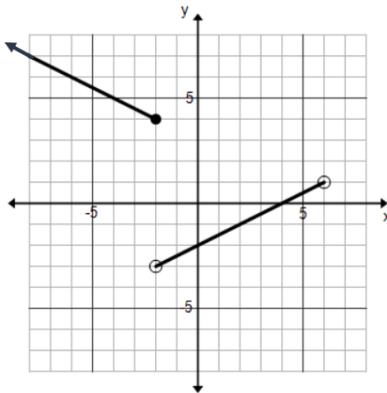
Is the function continuous?

End Behavior

## Where to Begin and End

For each function below, state the domain and range, end behavior, and continuity.

1.



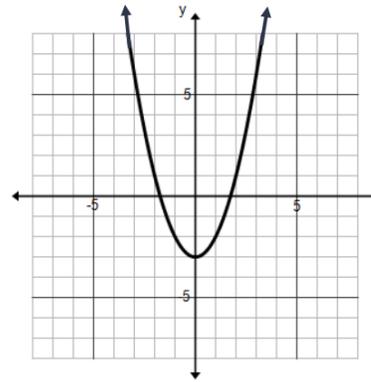
Domain: \_\_\_\_\_

Range: \_\_\_\_\_

End Behavior \_\_\_\_\_

\_\_\_\_\_

2.



Domain: \_\_\_\_\_

Range: \_\_\_\_\_

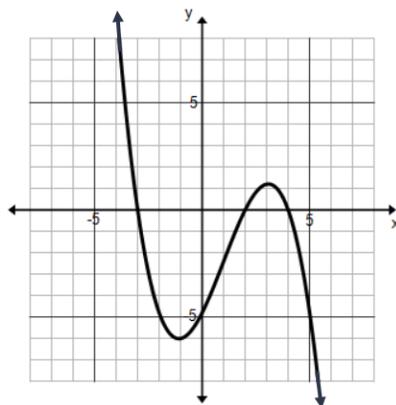
End Behavior \_\_\_\_\_

\_\_\_\_\_

Describe the continuity of the function:

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3.



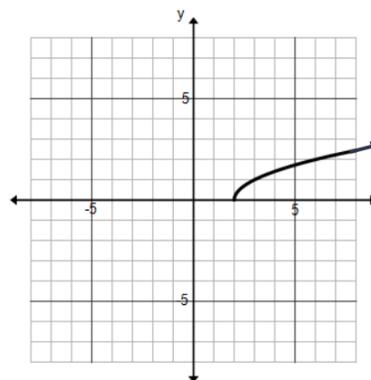
Domain: \_\_\_\_\_

Range: \_\_\_\_\_

End Behavior \_\_\_\_\_

\_\_\_\_\_

4.



Domain: \_\_\_\_\_

Range: \_\_\_\_\_

End Behavior \_\_\_\_\_

\_\_\_\_\_

Describe the continuity of the function:

Describe the continuity of the function:

5.  $f(x) = \frac{2x}{x-1}$

Domain: \_\_\_\_\_  
Range: \_\_\_\_\_  
End  
Behavior \_\_\_\_\_  
\_\_\_\_\_

Describe the continuity of the function:

6.  $f(x) = -3x + 5$

Domain: \_\_\_\_\_  
Range: \_\_\_\_\_  
End  
Behavior \_\_\_\_\_  
\_\_\_\_\_

Describe the continuity of the function:

7.  $f(x) = (x + 3)^2$

Domain: \_\_\_\_\_  
Range: \_\_\_\_\_  
End  
Behavior \_\_\_\_\_  
\_\_\_\_\_

Describe the continuity of the function:

8.  $f(x) = -2x^2 - 2$

Domain: \_\_\_\_\_  
Range: \_\_\_\_\_  
End  
Behavior \_\_\_\_\_  
\_\_\_\_\_

Describe the continuity of the function:

9.  $f(x) = x^3 + 6x^2 + 9x$

Domain: \_\_\_\_\_  
Range: \_\_\_\_\_  
End  
Behavior \_\_\_\_\_  
\_\_\_\_\_

Describe the continuity of the function:

10.  $f(x) = \left(\frac{1}{3}\right)^x + 2$

Domain: \_\_\_\_\_  
Range: \_\_\_\_\_  
End  
Behavior \_\_\_\_\_  
\_\_\_\_\_

Describe the continuity of the function:

## Domain and Range

For each function below, graph the function, state the domain and range, end behavior and continuity.

Function	Graph	Domain and Range	End Behavior	Continuity
1. $f(x) = \begin{cases} -3, & x \leq -4 \\ x+1, & x > -4 \end{cases}$		Domain:  Range:		
2. $y = x^2 + 3$		Domain:  Range:		
3. $g(x) = \frac{x}{x+5}$		Domain:  Range:		
4. $m(x) = 3^x - 1$		Domain:  Range:		
5. $y = \left(\frac{1}{2}\right)^x$		Domain:  Range:		