

Angles in Polygons

Strand: Polygons and Circles

Topic: Exploring angles in polygons

Primary SOL: G.10 The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the

- sum of the interior and/or exterior angles;
- measure of an interior and/or exterior angle; and
- number of sides of a regular polygon.

Related SOL: G.9

Materials

- Angles in Polygons activity sheet (attached)
- Tessellations and Angle Measures activity sheet (attached)
- Dynamic geometry software package
- Pattern blocks, for tessellations (optional)
- Patty paper

Vocabulary

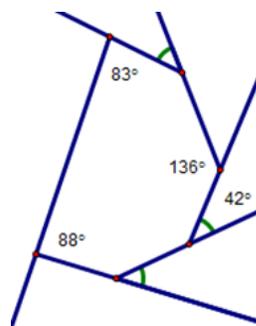
area, array, concave polygon, convex polygon, diagonal, exterior angle, factor, interior angle, irregular, n-gon, polygon, regular, regular tessellation, tessellate, tessellation, vertex

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Have students work in pairs to complete the Angles in Polygons activity sheet. Students will need to be told about “*n*-gon” when they get to the second page. Each student should record his/her own findings. Have students discuss their findings with their partners. Discuss the findings as a whole group.
2. Have students work in pairs to complete the Tessellations and Angle Measures activity sheet. Have students discuss their findings with their partners. Discuss the findings as a whole group.

Assessment

- **Questions**
 - What is wrong with the diagram on the right?



- How are an interior and an exterior angle at the same vertex related?

- How can you find the sum of the measures of the interior angles of a convex 102-gon without using the formula?
- If four of the angles of a pentagon measure 80 degrees, 90 degrees, 100 degrees, and 110 degrees, what is the measure of the fifth angle? Explain your reasoning.
- Which is greater, the measure of an exterior angle of a regular triangle or the measure of an interior angle of a regular pentagon? Explain your reasoning.
- **Journal/writing prompts**
 - Complete a journal entry summarizing your investigations.
 - Write directions for how to find the measure of an interior angle of a regular polygon.
 - If you are given the measure of an exterior (or interior) angle of a regular polygon, explain how to determine how many sides the polygon has.
 - Describe a practical situation that uses angles of polygons.
- **Other Assessments**
 - Have students explain and demonstrate why a given regular polygon can or cannot be used to tessellate a plane.
 - Have groups of students create “fragments” of regular polygons made from found materials (e.g., paper, fabric, craft foam, plastic) that are missing some of the angles and sides. Have groups swap fragments and determine how many sides the regular polygons had by measuring the interior angles.

Extensions and Connections

- Have students investigate irregular tessellations.
- Have students create a tessellations.
- Have students investigate tessellations in art, construction, and science.
- Virtual manipulatives demonstrating interior and exterior angles of polygons can be found online.
- Invite an artist or architect to the class to discuss the use of polygons and transformations in art or architecture.

Strategies for Differentiation

- Have students use auditory instructions to assist with the directions for the dynamic geometry software.
- Have students use 3-D models of polygons.
- Have students use a reflective transparent math geometry tool to explore and construct reflections for tessellations.
- Have students build a math glossary or folded graphic organizer with definitions and examples.
- Have students create a comparison table of the transformations used to make tessellations comparing the orientations and size of the images.
- Modify the table by eliminating rows or columns.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Angles in Polygons

Name _____ Date _____

Answer the questions, and complete the table.

Part 1: Interior Angles in Polygons

1. Use your book or other reference to complete the column titled *Name of Polygon* in the table. (Note that the table is two pages.) A polygon with n sides is called an n -gon.
2. What is the sum of the measures of the interior angles of a triangle? _____
3. We want to be able to find the sum of the measures of the interior angles of *any* convex polygon. Let's look at a quadrilateral. Draw a quadrilateral. Draw one diagonal.

How many triangles do you have? _____ How many degrees in each triangle? _____
What is the sum of the measures of the interior angles in a convex quadrilateral? _____

4. Repeat for a pentagon. (Draw a convex pentagon. Draw two diagonals *from the same vertex*.)

How many triangles do you have? _____ How many degrees in each triangle? _____
What is the sum of the measures of the interior angles in a convex pentagon? _____

5. Repeat No. 4, adding a side until you find patterns for the number of triangles and the sum of the measures of the interior angles.
6. Complete the table columns titled *# of Δ s* and *Sum of Interior Angles*.
7. If the sum of the measures of the interior angles of a triangle is 180 degrees, how large are each of the three congruent angles in a regular (equilateral, equiangular) triangle?
_____ How can you use the sum (180 degrees) and the number of sides (three) to get this?

8. What is another name for a regular quadrilateral? _____ What is the sum of the measures of the interior angles of a convex quadrilateral? (See No. 3) _____ Use this sum and the number of congruent angles in a regular quadrilateral to find the measure of each angle in a regular quadrilateral. _____

9. Repeat No. 8, adding a side until you find a pattern for the measure of each interior angle of a regular polygon.
10. Complete the table column titled *Each Interior Angle of Regular n-gon*.

Part 2: Exterior Angles in Polygons (using a dynamic geometry software package)

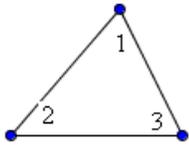
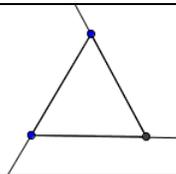
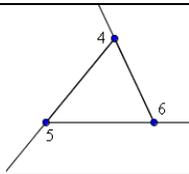
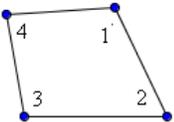
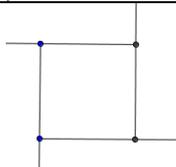
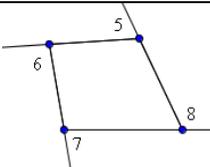
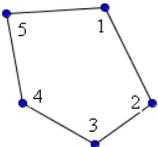
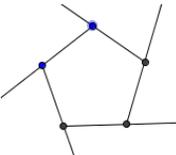
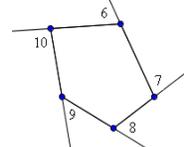
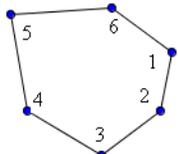
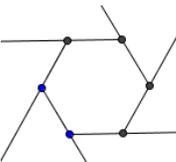
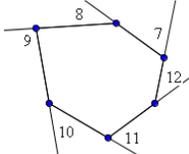
An exterior angle of a polygon is formed by extending a side of the polygon (into a ray). We want to be able to find the sum of the measures of the exterior angles of *any* convex polygon (if one exterior angle is drawn at every vertex.)

11. Using a dynamic geometry software package, draw a ray and name it \overrightarrow{AB} . Remember A is the endpoint of the ray and B is any other point on the ray. Now create another ray with endpoint at B . Name the point on this ray C , so the ray is \overrightarrow{BC} . Next create ray \overrightarrow{CA} . You now have a triangle with an exterior angle at each vertex. (*Note: If a dynamic geometry software package is not available, have the student draw a triangle or trace a pattern block.*)
12. Create a point on each of the three rays, *outside* the triangle. Measure each of the three exterior angles. Compute the sum. (You may be able to do this using the dynamic geometry software package without typing in the numbers.)
13. Move points A , B , and C , and recompute the sum. What do you notice? (*Note: If a dynamic geometry software package is not available, students could compare their answers to the answers of other students or teacher could create a class chart.*)
14. Write a conjecture (prediction) about the sum of the measures of the exterior angles of a triangle.
15. Open a new file. Repeat No. 11, adding an extra ray to create a quadrilateral. (Create \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} , \overrightarrow{DA} .) Add an extra point outside the polygon on each of the four rays, measure each of the exterior angles, and compute the sum.
16. Move the points to change the angle measures and recompute the sum. What do you notice?
17. Write a conjecture (prediction) about the sum of the measures of the exterior angles of a quadrilateral.
18. Repeat Nos. 15 and 16 for a pentagon. Continue, adding sides to your polygons, until you notice a pattern. Complete the table column titled *Sum of Exterior Angles*.
19. What is the sum of the measures of the exterior angles of a triangle? _____ (See No. 14) How can you use the sum and the number of angles (three) to get the measure of each of the three congruent *exterior* angles in a regular (equilateral, equiangular) triangle?
20. What is the measure of any exterior angle of a regular triangle? (You can look at the diagram in column *Regular n-gon* of the table.) _____

Mathematics Instructional Plan – Geometry

21. Use the sum of the measures of the exterior angles of a quadrilateral (see No. 17) and the number of angles to find the measure of each exterior angle of a regular quadrilateral.

22. Find a pattern, and complete the table column titled *Each Exterior Angle of Regular n-gon*.

n -gon	Formula →	$n - 2$	*		*		*	
Name of Polygon	n (# of sides)	# of Δ s	Convex n -gon	Sum of Interior Angles	Regular n -gon	Each Interior Angle of Regular n -gon	Convex n -gon	Sum of Exterior Angles
	3							
	4							
	5							
	6							
	7		*		*		*	
	8		*		*		*	
	9		*		*		*	
	10		*		*		*	

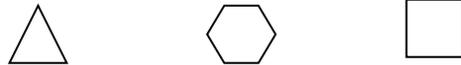
Name of polygon	n (# of sides)	# of Δ s	Convex n -gon	Sum of the interior angles	Regular n -gon	Each interior angle of regular n -gon	Convex n -gon
	11		*		*		*
	12		*		*		*
	15		*		*		*
	20		*		*		*
	100		*		*		*
	n (formula)		*		*		*

Tessellations and Angle Measures

Name _____ Date _____

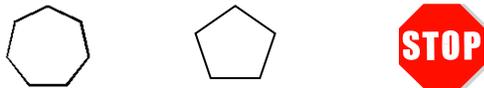
A tessellation is created by covering a plane with congruent (identical) shapes, without any overlaps or gaps. To tessellate means to create such a covering. Use patty paper to create tessellations or show that a shape will not tessellate. To use patty paper, carefully trace the shape, apply a transformation (e.g., rotation, translation, etc.), and trace again.

- Given the following regular polygons, demonstrate how each would tessellate a plane, using pattern blocks or patty paper.



- For each of the tessellations above, look at the vertices where several polygons meet. What is the sum of the angles around each vertex?

- For each of the following regular polygons, demonstrate how each would tessellate a plane, or show why the polygons do not tessellate.



- For your diagrams for each of the polygons above, look at a vertex where several polygons meet. What is the sum of the angles around each vertex?

- Complete the table below.

Regular Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon
Measure of one interior angle						
Sum of angles around a vertex						
Does it tessellate?						

- Explain how you can tell whether a regular polygon can tessellate a plane.

- Explain why other regular polygons cannot be used to create a tessellation.

8. Semiregular tessellations contain two or more regular polygons around each common vertex. Create a floor pattern that contains a semiregular tessellation pattern. Justify that your pattern is a semiregular tessellation with mathematical calculations and reasoning.