

Similar Solids and Proportional Reasoning

Strand: Three-Dimensional Figures

Topic: Investigating similar solids and proportional reasoning

Primary SOL: G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include

- comparing ratios between lengths, perimeters, areas, and volumes of similar figures;
- determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;
- determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and
- solving problems, including practical problems, about similar geometric figures.

Related SOL: G.7, G.13

Materials

- Investigating Similar Solids: Part 1 activity sheet (attached)
- Investigating Similar Solids: Part 2 activity sheet (attached)
- Exploring Volume activity sheet (attached)
- Extension: Gulliver’s Travels (attached)
- Excerpt from *Gulliver’s Travels* (attached)
- Plastic unit cubes
- Graph paper, same grid size as cubes

Vocabulary

area, diameter, dilate, dilation, dimension, length, perimeter, proportional, ratio, scale factor, similar figures, surface area, volume

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Distribute the Investigating Similar Solids: Part 1 activity sheet, and have students work in pairs to complete it. Each student should record his/her own findings. Have students discuss the findings with their partners. Discuss the findings as a whole group.
2. Distribute the Investigating Similar Solids: Part 2 activity sheet, and have students work in pairs to complete it. Each student should record his/her own findings. Have students discuss the findings with their partners. Discuss the findings as a whole group.
3. Distribute the Exploring Volume activity sheet, and have students work in pairs to complete it. Each student should record his/her own findings. Have students discuss the findings with their partners. Discuss findings as a whole group. The problems in Exploring Volume could be assigned to groups to complete and display their work on poster paper. Other groups could do a gallery walk to view the problem-solving processes different groups used. Alternately, the Exploring Volume problems could be assigned to different stations for groups of students to move through and complete.

Assessment

- **Questions**
 - If you double the height of a cylindrical paint can, what happens to the volume of the resulting cylinder/can? What happens to the area of the paint can label? Is the resulting cylinder/can similar to the original paint can? Explain.
 - If the scale factor of two similar cones is 2:3, and the volume of the small cone is 80 cubic inches, what is the volume of the larger cone? Explain and show your work.
 - If the surface areas of two similar solids are 4900π and 6400π , what is the ratio of the volumes? Explain and show your work.
 - Explain why all cubes are similar to each other.
- **Journal/writing prompts**
 - Complete a journal entry summarizing one of the activities.
 - Describe a real-world example that uses similar solids.
 - Write a practical problem and solution that uses similar solids.
- **Other Assessments**
 - Draw two spheres whose ratio of surface areas is 1:4.
 - Research the size of different balls used for various sports and determine two balls with a ratio of volumes 1:64.
 - Draw a rectangular prism with dimensions 2 x 3 x 4. Double the length, width, and height. Draw a picture to show how many of the smaller prisms you can pack into the larger prism. How is this related to the volumes of the two prisms?
 - Have students create a scale model of a cereal box using card stock, rulers, scissors, graph paper, and tape.

Extensions and Connections

- Invite a carpenter, builder, or architect to the class to demonstrate the various job applications that use surface area, volume, and proportional reasoning.
- Have students create nets of similar solids. (This can be done using graph paper and drawing tools or using a dynamic geometry software package to create a net and dilate it.)
- Have students read the excerpt from *Gulliver's Travels* at the end of this lesson. Lesson notes and supplementary questions are provided as well.
- Have students read the poem "One Inch Tall" by Shel Silverstein. Have students determine if one of the statements in the poem is true. (For example, could you walk beneath the door?)

Strategies for Differentiation

- When instruction is presented orally, provide a visual component to support the activity sheet. Use colored markers or chalk, transparencies or slides to present the information in small chunks. Provide a visual for each piece of oral information.

Mathematics Instructional Plan – Geometry

- Have students build the figures in Similar Solids: Part 1 using cubes before drawing them and completing the tables.
- Use isometric graph paper to help students visualize the three-dimensional component.
- Use colors to mark corresponding parts.
- Put proportions into words, like analogies, such as
$$\frac{\text{radius}}{\text{volume}} = \frac{\text{radius}}{\text{volume}} \text{ or } \frac{\text{small}}{\text{small}} = \frac{\text{medium}}{\text{medium}} = \frac{\text{large}}{\text{large}}.$$
- Have students use the rules $\frac{\text{small}}{\text{big}} = \frac{\text{small}}{\text{big}}$ and $\frac{\text{big}}{\text{small}} = \frac{\text{big}}{\text{small}}$.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Investigating Similar Solids: Part 1

Name _____ Date _____

1. On the graph paper, draw a rectangle (not a square) with length and width between 2 and 5 units.
2. Leaving at least three rows between the two rectangles, draw another rectangle with the length and width twice the length and width of the *original* rectangle.
3. Leaving at least three rows between the rectangle and the other rectangles, draw a third rectangle with the length and width three times the length and width of the *original* rectangle.
4. Complete the table below.

	Small Rectangle	Medium Rectangle	Large Rectangle
Length			
Width			
Perimeter			
Area			

5. How many of the small rectangles fit in the medium rectangle? _____
6. How many of the small rectangles fit in the large rectangle? _____
7. Compute the ratios of the three rectangles' lengths, perimeters, and areas using the data above.

Complete the table below. *Be sure to reduce all ratios!*

Ratios:	Small: Medium	Small: Large	Medium: Large
Length			
Perimeter			
Area			

8. How are the ratios of the areas related to your answers to Nos. 2 and 3?

9. Use cubes to build a small rectangular prism with the length and width the same as the small rectangle and with a height of 1 unit. How many rectangles do you need? _____ Sketch this prism on your small rectangle by adding diagonals as shown in figure 1.
10. If you have enough cubes, build a medium rectangular prism with the length and width the same as the medium rectangle and with a height of 2 units. How many rectangles do you need? _____ Sketch this prism on your medium rectangle by adding diagonals as shown in figure 1, only two deep.
11. If you have enough cubes, build a large rectangular prism with the length and width the same as the large rectangle and with a height of 3 units. How many rectangles do you need? _____ Sketch this prism on your large rectangle by adding diagonals as shown in figure 1, only three deep.

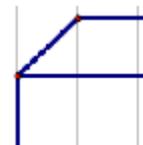


figure 1

12. Complete the rest of the table below.

	Small Rectangular Prism	Medium Rectangular Prism	Large Rectangular Prism
Height	1	2	3
Surface Area			
Volume			

13. Compare the volumes of the prisms to your answers to Nos. 7, 8, and 9. Explain.

14. Similar solids are solids of the same type (i.e., spheres and rectangular prisms) that have proportional linear measures (i.e., length, width, height, perimeter, and radius). Are your three rectangular prisms similar? How do you know?

15. How many of the small rectangular prisms fit in the medium rectangular prism? _____

16. How many of the small rectangular prisms fit in the large rectangular prism? _____

17. Compute the ratios of the three rectangular prisms' heights, surface areas, and volumes using the data above. Complete the table below. *Be sure to reduce all ratios!*

Ratios:	Small: Medium	Small: Large	Medium: Large
Height			
Surface Area			
Volume			

18. How are the ratios of the volumes related to your answers to Nos. 14 and 15?

19. What is the scale factor of the small rectangular prism to the medium rectangular prism?

20. What is the scale factor of the small rectangular prism to the large rectangular prism?

21. How are the scale factors (No. 16) related to the corresponding ratios of surface areas and volumes?

Investigating Similar Solids: Part 2

Name _____ Date _____

You must show your work, including proportions or other equations where appropriate (Nos. 4–8). Do not round your answers. (i.e., $1,125\sqrt{3\pi} \text{ cm}^3$, rather than 6121.6 cm^3)

The ratio of the heights of two similar cones is 2:5. The radius of the smaller cone is 6 centimeters.

1. What is the scale factor of the two cones?
2. What is the radius of the larger cone?
3. What is the ratio of the lateral areas?
4. The lateral area of the smaller cone is 72π square centimeters. What is the lateral area of the larger cone? (Hint: Use No. 3)
5. What is the ratio of the volumes?
6. The volume of the larger cone is $1,125\sqrt{3\pi}$ cubic centimeters. What is the volume of the smaller cone? (Hint: Use No. 5)
7. Find the slant heights and heights of the two cones. (You may already have found one or more of these.)
8. Find the angle x formed by a radius and the sides of the cones.
9. Complete the table below.

	r	L.A.	V	l	h
Small Cone		72π			
Large Cone			$1,125\sqrt{3\pi}$		
Ratio	2:5				

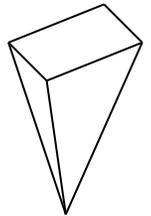
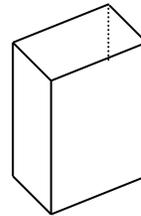
10. Compute the volumes of the following solids:
 - a) A cylinder with height r and radius r
 - b) A hemisphere with radius r
 - c) A cone with height r and radius r
11. Draw diagrams of the three figures. Try to draw them so they have the same radius and height. Compare the volumes. How many cones of water would it take to fill the cylinder? The hemisphere?

Exploring Volume

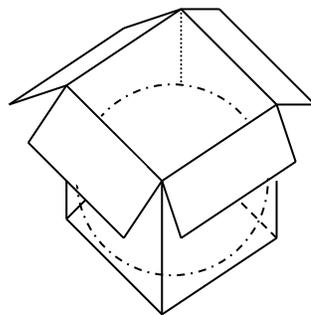
Name _____ Date _____

1. Which will carry the most water in a given length: two pipes with one having a 3 centimeter radius and the other a 4 centimeter radius, or one pipe with a 5 centimeter radius? Explain.
2. A company delivers 36 cartons of paper to your school. Each carton measures 40 x 30 x 25 centimeters. Is it possible to fit all cartons in an empty storage closet 1 x 1 x 2 meter? Justify your conclusion with a visual explanation.
3. You have studied the pyramids and want to make a scale model of a pyramid with a square base and sides that are isosceles triangles. How much clay is required if the base of the actual pyramid is 30 meters on each side and the height of the pyramid is 30 meters? Your scale is 1 centimeter = 15 meters.

4. A movie theater decides to change the shape of its popcorn holder from a rectangular box to a pyramidal box. The tops of both boxes are the same and the height remains the same. If the rectangular bag of popcorn cost \$4.00, what is a fair price for the new box?



5. A manufacturer of globes that are approximately 1 meter in diameter packs the globes in 1-cubic-meter boxes for shipping. How much packing material (foam peanuts) is needed for a shipment of 100 globes?



6. Take two sheets of paper the same size. Roll one sheet vertically and tape to form a right circular cylinder. Roll the second sheet horizontally, and tape it to form a second right circular cylinder. Tape each cylinder so that there is no overlap of paper (i.e., the edges should meet exactly). If each cylinder were filled with popcorn, would they contain the same amount? Explain and justify your answer.

Extension: Gulliver’s Travels

Teacher Notes

Materials

- Plastic linking cubes
- Excerpt from *Gulliver’s Travels* (public domain)
- Calculators
- Objects like those in Gulliver’s pockets (handkerchief, comb, folded and tied paper, snuff box, pocket watch, knife, and other objects described)

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Have students read the excerpt from *Gulliver’s Travels*. Take time to discuss the objects that are described there. Help students identify at least five of the objects described.
2. Using the objects like those found in Gulliver’s pockets, have students estimate the size of the Lilliputians. Have students measure the comb, handkerchief, and other objects, and help them to investigate the proportional relationships.
3. At an earlier point in the story, the Lilliputians feed Gulliver and are amazed by the amount of food he consumes. This is related to his volume and overall size as much as to his height. While the volume of a human is difficult to calculate, the volume of cubes and spheres is not. Use these shapes to investigate the relationship between changes of side length/diameter and surface area and volume.

Assessment

- **Journal/Writing Prompts**
 - Would you prefer to visit Lilliput or to be a Lilliputian visiting Gulliver’s England? Explain.
- **Other**
 - Have students estimate what size a modern device (such as a cellphone or car) would have to be for a Lilliputian to use it as Gulliver would. Allow students to choose their device. As an extension, students could investigate whether modern technology would allow their device to be made that small. (For example, can cellphone memory be made that small?)

Excerpt from Gulliver’s Travels

By Jonathan Swift
(public domain)

Gulliver’s ship was caught in a storm. He swam to safety on a mysterious island called Lilliput. He was captured and bound, taken to the Emperor’s castle, and presented to the Emperor and Empress. When he was released from his rope bindings, the Lilliputians wanted to ensure that he had no weapons. This is the inventory of the contents of his pockets. (From part I, chapter II)

Imprimis. In the right coat-pocket of the Great Man-Mountain (the Lilliputian’s name for Gulliver) after the strictest search, we found only one great piece of course cloth, large enough to be a foot-cloth for your Majesty’s chief room of state. In the left pocket, we saw a huge silver chest, with a cover of the same metal, which we, the searchers, were not able to lift. We desired it should be opened, and one of us, stepping into it, found himself up to the midleg in a sort of dust, some part whereof, flying up to our faces, set us both a sneezing for several times together. In his right waistcoat-pocket, we found a prodigious bundle of white thin substances, folded one over another, about the bigness of three men, tied with a strong cable and marked with black figures; which we humbly conceive to be writings, every letter almost half as large as the palm of our hand. In the left, there was a sort of engine (a term used for any mechanical device), from the back of which were extended twenty long poles, resembling the palisades before your Majesty’s court; wherewith we conjecture the Man-Mountain combs his head, for we did not always trouble him with questions, because we found it a great difficulty to make him understand us. In the large pocket on the right side of his middle cover (so I translate the word ranfu-lo, by which they meant breeches) we saw a hollow pillar of iron, about the length of a man, fastened to a strong piece of timber, larger than the pillar; and upon one side of the pillar were huge pieces of iron sticking out, cut into strange figures, which we know not what to make of. In the left pocket, another engine of the same kind. In the smaller pocket on the right side, were several round flat pieces of white and red metal, of different bulk; some of the white, which seemed to be silver, were so large and heavy, that my comrade and I could hardly lift them. In the left pocket were two black pillars, irregularly shaped: we could not, without difficulty, reach the top of them as we stood at the bottom of his pocket. One of them was covered, and seemed all of a piece; but at the upper end of the other, there appeared a white round substance, about twice the bigness of our heads. Within each of these were enclosed a prodigious plate of steel; which, by our orders, we obliged him to show us, because we apprehended they might be dangerous engines. He took them out of their cases, and told us, that in his own country his practice was to shave his beard with one of these, and to cut his meat with the other. There were two pockets which we could not enter: these he called his fobs; there were two large slits cut into the top of his middle cover, but squeezed closed by the pressure of his belly. Out of the right fob hung a great silver chain, with a wonderful kind of engine at the bottom. We directed him to draw out whatever was at the end of that chain; which appeared to be a

globe, half silver, and half of some transparent metal: for on the transparent side we saw strange figures circularly drawn, and thought we could touch them, until we found our fingers stopped with that lucid substance. He put this engine to our ears, which made an incessant noise like that of a watermill. And we conjecture it is either some unknown animal, or the god that he worships; but we are more inclined to the latter opinion, because he assured us (if we understood him right, for he expressed himself very imperfectly), that he seldom did anything without consulting it. He called it his oracle, and said that it pointed out the time for every action of his life. From the left fob he took out a net almost large enough for a fisherman, but contrived to open and shut like a purse, and served him for the same use: we found therein several massy pieces of yellow metal, which, if they be of real gold, must be of immense value.

Having thus, in obedience to your Majesty's commands, diligently searched all his pockets, we observed a girdle about his waist made of the hide of some prodigious animal; from which, on the left side, hung a sword of the length of five men, and on the right, a bag or pouch divided into two cells, each cell capable of holding three of your Majesty's subjects. In one of these cells were several gloves or balls of a most ponderous metal, about the bigness of our heads, and required a strong hand to lift them: the other cell contained a heap of certain black grains, but of no great bulk or weight, for we could hold above fifty of them in the palms of our hands.

This is an exact inventory of what we found about the body of the Man-Mountain, who used us with great civility, and due respect to your Majesty's commission. Signed and sealed on the fourth day of the eighty-ninth moon of you Majesty's auspicious reign.

CLEFREN FRELOCK, MARSİ FRELOCK