Inductive and Deductive Reasoning

Strand: Reasoning, Lines, and Transformations

Topic: Practicing inductive and deductive reasoning strategies

Primary SOL: G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

c) determining the validity of a logical argument.

Related SOL: G.2a, G.6, G.7, G.9

Materials

- Inductive and Deductive Reasoning: Part 1 activity sheet (attached)
- Inductive and Deductive Reasoning: Part 2 activity sheet (attached)
- Algebraic Properties of Equality activity sheet (attached)

Vocabulary

conclusion, conditional statement, conjecture, deductive reasoning, disprove, hypothesis, inductive reasoning, logical argument, observation, prove, valid argument, verify

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Distribute the Inductive and Deductive Reasoning activity sheets (Part 1 and Part 2). Review the vocabulary for this lesson. Have students work in pairs or small groups to complete them.
2. Distribute the Algebraic Properties of Equality activity sheet, and have students work in pairs or small groups to complete it.

Assessment

- **Questions**
  - How could you disprove the statement, "All fish fly"?
  - What is the next term in the sequence 1, 2, 3, 8, 16, ________? Did you use inductive or deductive reasoning? Explain.
  - The definition of vertical angles says, "If two angles are vertical, then they are congruent." $\angle 1$ and $\angle 2$ are vertical. What can you conclude? What type of reasoning did you use (inductive or deductive)?
  - Prove or disprove the following conjecture:
    Conjecture: For all real numbers x, the expression x^2 is greater than or equal to x.
- **Journal/writing prompts**
  - Have students complete a journal entry summarizing inductive and deductive reasoning strategies.

  - Have students write about a time when they used inductive or deductive reasoning. Students should explain how they know they used inductive or deductive reasoning.
  - Does inductive reasoning always result in a true conjecture? Explain.

- **Other Assessments**
  - Give an example of correct deductive reasoning using conditional statements. Explain why the reasoning is correct.
  - Give an example of faulty reasoning using conditional statements.

Extensions and Connections (for all students)

- Have students investigate Lewis Carroll's logic puzzles.
- Have students solve the logic puzzle from J.K. Rowling's *Harry Potter and the Sorcerer's Stone,* 1998, p. 285.
- Invite a journalist or political analyst to visit the class. Ask the guest speaker to explain the relationships among facts, trends, and educated guesses.
- Complete the conjecture based on the patterns you observe:
  Conjecture: The product of a number $(n - 1)$and the number $(n + 1)$ is always equal to _______.

Strategies for Differentiation

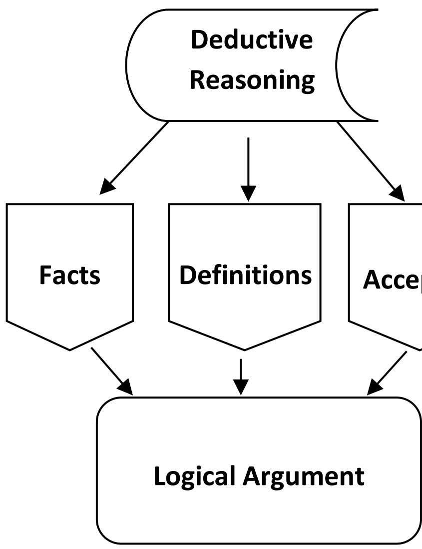
- Have students use presentation software to create presentations of the vocabulary terms. This will allow them to provide their own visual cues and props to reinforce the learning process.
- Post the properties in the room with the statements.
- Have students write the vocabulary on index cards and practice their understanding with one another before and after the lesson.
- Have students draw illustrations to represent the mathematical properties.
- Have students use a diagram to illustrate the information (each student could have his/her own diagram).
- Have students create a comic strip of someone using inductive reasoning or deductive reasoning.
- Use demonstration tool (e.g., document camera, digital display) to illustrate the steps in deductive and inductive reasoning.
- Provide visual representation to compare deductive and inductive reasoning. Explain to the class that the inductive process begins with few facts but ends with many possible conclusions. Then explain that the deductive process begins with many facts but ends with few possible conclusions.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

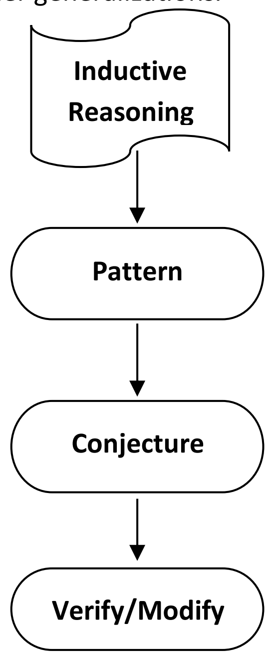
Inductive and Deductive Reasoning: Part 1

Name ______________________________ **Date** ____________________________

Deductive reasoning is a method of reasoning that uses logic to draw conclusions based on definitions, postulates, and theorems.



Inductive reasoning works from more specific observations to broader generalizations.



Example of Deductive Reasoning	Example of Inductive Reasoning
• Tom knows that if he misses the practice the day before a game, then he will not be a starting player in the game. • Tom misses practice Tuesday. • *Conclusion:* Tom will not be a starting player in the game Wednesday.	• *Observation:* Mia came to class late this morning. • *Observation:* Mia's hair was uncombed. • *Prior Experience:* Mia is fussy about her hair. • *Conclusion:* Mia overslept.

A. Complete the following conjectures based on the patterns you observe in specific cases:

Step 1: List all odd numbers from 1-20.

Step 2: Make a table that sums the first two numbers, then the second two numbers.

Step 3: Conjecture: The sum of any two odd numbers is ________.

1 + 3 = 4	7 + 11 = 18
3 + 5 = 8	13 + 15 = 28
5 + 7 = 12	15 + 17 = 32

B. Complete the following conjectures based on the patterns you observe in specific cases:

Step 1: List all odd numbers from 1-20. Make a table that multiplies the first two numbers, then the second two numbers.

1 x 3 = 3	7 x 11 = 77
3 x 5 = 15	13 x 15 = 195
5 x 7 = 35	15 x 17 = 255

Step 2: Conjecture: The product of any two odd numbers is _____.

Inductive and Deductive Reasoning: Part 2

Name ________________________________ **Date** ___________________________

1. John always listens to his favorite radio station, an oldies station, when he drives his car. Every morning he listens to his radio on the way to work. On Monday, when he turns on his car radio, it is playing country music. Make a list of valid conjectures (predictions) to explain why his radio is playing different music.
2. ∠M is obtuse. Make a list of conjectures based on that information.
3. Based the table below, Marina concluded that when one of the two addends is negative, the sum is always negative. Write a counterexample for her conjecture.

Addends	Sum	
−8	−10	−18
−17	−5	−22
15	−23	−8
−26	22	−4

4. Construct a conjecture based on patterns you observe below.

Case 1: Christine notices that her friend, Endia, is absent from school every first Monday of the month. It is the first Monday of the month. What can she deduce?	Case 2: Mrs. Batten notices that whenever Johnny is nervous he twirls his pencil. Johnny is twirling his pencil. What can Mrs. Batten deduce?
Case 3: Whenever Usain Bolt wears his golden shoes, he wins the 400-meter race. If he wins this race, then he is the fastest man in the world. Usain Bolt wears his golden shoes to the 2010 Olympics. Make a conjecture.	Case 4: At the Wallops Flight Facility in Virginia, a rocket cannot be launched in the rain. It is raining. Make a conjecture.

Algebraic Properties of Equality

The algebraic properties of equality can be used to solve and justify $5x - 18 = 3x + 2$, by writing a reason for each step, as shown in the table below.

Statement	Reason
$5x - 18 = 3x + 2$	Given
$2x - 18 = 2$	Subtraction Property of Equality
$2x = 20$	Addition Property of Equality
$x = 10$	Division Property of Equality

Using a table like the one above, solve each of the following equations, and state a reason for each step, using the properties contained below.

1. $-2(-w + 3) = 15$
2. $p - 1 = 6$
3. $2r - 7 = 9$
4. $3(2t + 9) = 30$
5. Given $3(4v - 1) - 8v = 17$, prove $v = 5$.

Addition Property of Equality	**If $a = b$, then $a + c = b + c$**
Subtraction Property of Equality	**If $a = b$, then $a - c = b - c$**
Multiplication Property of Equality	**If $a = b$, then $ac = bc$**
Division Property of Equality	**If $a = b$ and $c \neq 0$, then $a \div c = b \div c$**
Reflexive Property	**$a = a$**
Symmetric Property	**If $a = b$, then $b = a$**
Transitive Property	**If $a = b$ and $b = c$, then $a = c$**
Substitution Property	**If $a = b$, then a can be substituted for b in any equation or expression.**
Distributive Property	**$a(b + c) = ab + ac$**

Note: a, b, and c are real numbers.