Circle Fractions: Comparing and Ordering with Benchmarks

Strand: Number and Number Sense

Topic: Comparing, ordering, and representing fractions and mixed numbers using an area model.

Primary SOL: 4.2 The student will
a) compare and order fractions and mixed numbers, with and without models.*

** On the state assessment, items measuring this objective are assessed without the use of a calculator.*

Related SOL: 4.2b

Materials

- Fraction Circles (To Fourths) activity sheet (attached; one per student)
- Fraction Circles (Fifths to Eighths) activity sheet (attached; one per student)
- Fraction Circles (Ninths to Twelfths) activity sheet (attached; one per student)
- To Be Half, or Not to be Half….That is the Comparison activity sheet (attached)
- Benchmark Fractions Organizer activity sheet (attached)
- Fraction Sets (attached)

Vocabulary

benchmark, compare, denominator, equal, equal to, equivalent, fraction, greater than, improper fraction, less than, mixed number, numerator, order, proper fraction

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Distribute Fraction Circles activity sheets to each student. Review the importance of establishing the whole. Ask students to take out a complete circle and ask what they would call this part of the fraction set. Ask what number would be associated with this piece, and establish that 1 is the numeral that could name the piece. It is important to define the whole so that the students will be able to identify the part-to-whole relationship of the fractional parts within the area model represented by the parts of the whole circle.
2. Then hold up the one-half piece and ask the students what number could be assigned to it. Once the students have identified that it is $\frac{1}{2}$, ask students to explain how they know. Some sample responses should include: two halves equal a whole circle in the set, or if both of the one-half pieces are put together, they make the whole circle piece.
3. Draw a number line and label the 0 and the 1. Ask a volunteer to come up and show where the fraction $\frac{1}{2}$ should be placed. Ask the class for thumbs-up if they agree, thumbs-down if they do not, or thumbs sideways if they are not sure. If all thumbs are

not up, facilitate a discussion to clarify where $\frac{1}{2}$ should be placed and why. Then ask a volunteer to come up and show where $\frac{2}{2}$ should be placed and why. Ask for students to show thumbs. If all thumbs are not up, facilitate a discussion to clarify that $\frac{2}{2}$ is equal to 1, so they are located at the same point. To prepare students for the next activity, ask whether anyone can identify a fraction that would be located between zero and $\frac{1}{2}$; accept several answers without discussion. Then ask whether anyone knows a fraction that would be located between $\frac{1}{2}$ and 1; accept several answers without discussion. Then let students know that next they are going to investigate what kind of fractions are between zero and $\frac{1}{2}$ and between $\frac{1}{2}$ and 1.

4. In this activity the students, alone or in pairs, will use the fraction circles to identify other fractions and compare them to the benchmark of one-half that was defined in the beginning of the lesson. Introduce the term *benchmark*. Distribute the Benchmark Fractions Organizer to helps students create a model of a fraction and determine whether it less than $\frac{1}{2}$, equal to $\frac{1}{2}$, or more than $\frac{1}{2}$ and record the fraction in the appropriate column. They should find at least four fractions to record in each column. Remind students that they can use more than one piece from the fraction set to create a fraction and then record its name. Throughout the lesson, walk around to identify those students who are unable to correctly write and name the fractions based on the circle pieces. Pose questions to help them think through the task. Remind them to answer the three questions on the Benchmark Fractions Organizer to prepare for the class discussion.
5. When the students have completed the task, facilitate a class discussion based on the three questions. Begin by asking what they notice about the fractions equal to one-half. Listen for statements that focus on the relationship between the numerator and denominator. Included below are some of the relationships that the students may notice.
    - The numerator is half of the denominator.
    - When you double the numerator, it will equal the denominator.
    - When you divide the denominator by two, the quotient is the same value as the numerator.
    - If you multiply the numerator by two, the product will be the same value as the denominator.
    - If you subtract the numerator from the denominator, the difference will be the same value of the numerator.

    Ask students to think about the class discussion. Without using the fraction pieces, have students make up one or two fractions that belong in the equal to $\frac{1}{2}$ column. Then ask for some volunteers to share their fraction and have the class vote with their thumbs. Once students have identified the fractions that are equivalent to $\frac{1}{2}$ and have focused on the relationship between the numerator and denominator, ask the students whether those characteristics are always true for all fractions equal to one-half. End this

discussion by restating the relationship between the numerator and denominator if a fraction is equal to $\frac{1}{2}$.

6. Next, facilitate a discussion about what students noticed about the fractions in the less than $\frac{1}{2}$ column. Have a class discussion on the characteristics of the fractions that the students share. Included below are sample responses.

    - The numerator is less than $\frac{1}{2}$ of the denominator.
    - Two times the numerator is less than the denominator.
    - The fraction is less than the fraction equivalent to $\frac{1}{2}$.

    Ask students to think about the class discussion. Now, without using the fraction pieces, have students make up one or two fractions that belong in the less than $\frac{1}{2}$ column. Then ask for volunteers to share their fraction and have the class vote with their thumbs. Once the students have identified the fractions that are less than $\frac{1}{2}$ and have focused on the relationship between the numerator and denominator, ask the students whether those characteristics are always true for all fractions less than one-half. End this discussion by restating the relationship between the numerator and denominator if a fraction is less than $\frac{1}{2}$.

7. Last, have a class discussion on the characteristics of the fractions that are more than $\frac{1}{2}$. Included below are sample responses.

    - The numerator is more than $\frac{1}{2}$ of the denominator.
    - Two times the numerator is more than the denominator.
    - The numerator is close to the same value as the denominator.
    - The fraction is more than the fraction equivalent to $\frac{1}{2}$.
    - The fraction is close to the fraction that is equal to a whole.

    Ask students to think about the class discussion. Now, without using the fraction pieces, have students make up one or two fractions that belong in the more than $\frac{1}{2}$ column. Then ask for volunteers to share their fraction and have the class vote with their thumbs. Once the students have identified the fractions that are more than $\frac{1}{2}$ and have focused on the relationship between the numerator and denominator, ask the students whether those characteristics are always true for all fractions more than one-half. End this discussion by restating the relationship between the numerator and denominator if a fraction is less than $\frac{1}{2}$.

8. Have the students put away the fraction circle pieces and give them additional fractions to sort on the Benchmark Fractions Organizer. Write a set of fractions with denominators of 12 or less on the board that have not been identified with the fraction circles. Provide the students fractions with an odd denominator, such as $\frac{3}{7}$ or $\frac{8}{11}$, and an odd numerator, such as $\frac{7}{12}$. Circulate around the room and note any students with

confusions or misconceptions. Some students may have a difficult time figuring out the fractional equivalent to one-half when the denominator is 7. There are several different strategies and models you can use to determine the fraction equivalent to half when the denominator is an odd number. One strategy is to ask the students to tell you what is half of six and what is half of eight, using that information students should be able to determine half of seven. A linear model (such as a number line) or a pictorial area model (such as a square model) would also demonstrate that the $\frac{3}{7}$ is less than half. Have the same discussion with $\frac{8}{11}$ to show that it is more than half. When the class has completed the task, have them share answers and through discussion address any misconceptions.

9. Ask the students to think about the fractions $\frac{8}{5}$ and $1\frac{3}{4}$ and where they would be placed on the Benchmark Fractions Organizer. Listen to student responses that indicate the fractions are more than $\frac{1}{2}$. Then redirect students' attention back to the number line model from earlier in the lesson and ask if they think $\frac{1}{2}$ is a good benchmark to examine $\frac{8}{5}$ and $1\frac{3}{4}$. Though questioning, the students will bring out that both of these fractions are more than one whole (1) so that may be a better benchmark. Ask the students whether we can modify our benchmark organizer to include fractions that are more than a whole. Students should realize that we can include additional columns, such as equal to a whole, more than whole, or columns for fractions with any amount of wholes. Our columns should reflect the fractions that we are sorting or comparing.
10. Next, ask the students how they could use benchmarks of zero, $\frac{1}{2}$, or 1 when comparing or ordering fractions? Give the students a set of fractions for them to order. Have the students create their own benchmark chart that would best represent the fractions given below. (Note: The use of *more than four* fractions in the following activity allows students to utilize various benchmarks as a strategy for comparing and ordering fractions. Note: Students are only comparing one fraction at a time to the various benchmarks. Once fractions have been sorted, they do appear to be in order by size.)

$$\frac{3}{6}, \frac{8}{8}, \frac{1}{3}, \frac{5}{4}, \frac{6}{7}$$

Circulate as students are working, and choose several students you would like to have share their work. One example to look for is the following chart so that the class can discuss the value of the benchmarks.

Less than $\frac{1}{2}$	Equal to $\frac{1}{2}$	More than $\frac{1}{2}$	Equal to 1	More than 1

Some students will recognize that one fraction is more than a whole and one of is equal to one.

- Give the students another set of fractions (such as those listed below) so that once they are sorted (based on benchmarks) there will be more than one

fraction in a column. *In this activity, students are only comparing one fraction at a time to each of the benchmarks.*

$$\frac{7}{8}, \frac{1}{3}, \frac{2}{5}, \frac{4}{8}, \frac{6}{5}, \frac{2}{3}$$

- Then have students draw a number line with the 0, $\frac{1}{2}$, 1 indicated. Instruct the students to place the numbers from the benchmark table on the number line in the order of their size and distance from the benchmark. Students may need to use an additional strategy to order the fractions, such as looking at the distance from one- half. Students could also use their fraction circles, use pictorial representations, or find equivalent fractions or a common denominator to compare the fractions.
- For example, the fractions $\frac{1}{3}$ and $\frac{2}{5}$ are both less than half. We can find equivalent fractions to determine the distance from one-half. Knowing that $\frac{1}{3}$ is equal to $\frac{2}{6}$, students can then compare $\frac{2}{6}$ and $\frac{2}{5}$. Students need to be able to think about this as two sixths and two fifths, and since one-fifth is more than one-sixth, then two-fifths is more than two-sixths. The class discussions following the students work on the number line is critical in developing students' understanding about the relationships among numerators and denominators.

11. Last, have students work with a partner to complete the Fraction Sets activity sheet. Encourage the students to create their own benchmark charts or number lines to sort their fractions based on the benchmarks 0, $\frac{1}{2}$, and 1, and use strategies to order from least to greatest. Included are different sets that focus on benchmarks and using other strategies to compare. These strategies may include looking at the numerator and denominator, the distance from the whole or half, and finding equivalent fractions. Have the students share how they used these strategies to order the fractions.
    - Set 1: focusing on benchmarks and unit fractions
    - Set 2: focusing on benchmarks and like numerators
    - Set 3: focusing on benchmarks and fractions close to half
    - Set 4: focusing on benchmarks and improper fractions
    - Set 5: focusing on benchmarks and close to a whole
    - Set 6: focusing on benchmarks and fractions more than a whole

Assessment

- **Questions**
    - What is the numerator if the denominator is 12 and the fraction is equal to a half?
    - What is the denominator if the numerator is 4 and the fraction is equal to a half?
    - How far is $\frac{5}{11}$ from a half? Use pictures or words to explain.
    - Which fraction is closer to a half, $\frac{3}{7}$ or $\frac{4}{9}$? Use pictures or words to explain your answer.

- **Journal/writing prompt**
  - Use pictures and words to explain why $\frac{10}{11}$ is greater than $\frac{3}{4}$.
  - Put the following fractions in order from least to greatest: $\frac{7}{8}$, $\frac{3}{4}$, $\frac{2}{8}$, $\frac{1}{3}$. Write an explanation of your order.
- **Other Assessments**
  - Which is more, $\frac{1}{4}$ or $\frac{1}{5}$ of a pizza? How do you know?
  - Which is less, $\frac{1}{2}$ or $\frac{3}{4}$ of a bag of chips? How do you know?
  - Maria has a piece of rope that is $\frac{3}{4}$ of a foot long. Tomas has a piece of rope that is $\frac{1}{2}$ of a foot long. Mitch has a piece of a rope that is $\frac{1}{3}$ of a foot long. Who has the most rope? How do you know?
  - Ms. Crane noticed the following about her students on the playground Monday.

    $\frac{2}{5}$ of the students are wearing sweatshirts.

    $\frac{7}{10}$ of the children are wearing tennis shoes.

    $\frac{5}{12}$ of the children are wearing blue shirts.

    $\frac{2}{3}$ of the children are wearing a stocking hat.

    For each item of clothing, are more than half or less than half of the children wearing that item? How do you know?

Extensions and Connections

- Have students create a human number line with fractions on index cards. First define the whole and ask students whether there is another benchmark that would be helpful to locate. Identifying $\frac{1}{2}$ would be important when ordering the fractions on a number line. Next, give each student an index card with a fraction and have the student stand in the correct location on the human number line.
- Have students play the activity "To Be Half, or Not to Be Half…That is the Comparison" with a partner. This activity helps students become more familiar with fractions, using concrete models to determine if the fractional part is less than $\frac{1}{2}$, equal to $\frac{1}{2}$, or greater than $\frac{1}{2}$. Each player should have a Fraction Sorting Mat (attached), a set of the Fraction Cards (attached) and fraction manipulatives, such as fraction circles (attached) or fraction strips. Students should first cut the fraction cards apart, shuffle them, and place them face down in a stack. Player One will take a fraction card from the top of the deck and determine if the fraction is "less than $\frac{1}{2}$", "equal to $\frac{1}{2}$", or "greater than $\frac{1}{2}$" and place the fraction card in the appropriate section on the sorting mat. To prove that the card is in the correction location on the sorting mat, Player Two will use the fraction manipulatives to model the fraction on the card and compare it to a model of $\frac{1}{2}$. If Player Two agrees with Player One, then Player One will earn one point. If Player One

has placed the fraction card in the incorrect location on the sorting mat then he/she will not earn a point. Next, Player Two will select a fraction card from the deck and place the card in the correct location on the sorting mat. Player One will use the manipulatives to model the fraction on the card to determine if Player Two receives a point for placing the fraction card in the correction section on the sorting mat. Both players will continue to take turns until all of the cards have been sorted or time is up. Player with the most points wins.

Strategies for Differentiation

- Use concrete models for the activities as needed.
- Provide students with fraction strips that are closely related to the number line model.
- Use folding paper to show the relationship between fractions and equivalent fractions.
- Create anchor charts for the Benchmark Fractions Organizer to include a description of the relationship of the numerator and denominator, and a visual model.
- Identify three or more fractions between two fractions. For example: What are three fractions between $\frac{1}{2}$ and 1? Between $\frac{1}{2}$ and $\frac{3}{4}$?

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Benchmark Fractions Organizer

Use the fraction pieces from the circle fraction set to find fractions that are less than $\frac{1}{2}$, equal to $\frac{1}{2}$, and greater than $\frac{1}{2}$. You can use more than one piece of the fraction circle in the investigation. Record your fractions in the appropriate column below, try to find at least four fractions in each column. Then answer the questions below.

Less than $\frac{1}{2}$	Equal to $\frac{1}{2}$	More than $\frac{1}{2}$

1. Look at all the fractions equal to $\frac{1}{2}$. What do you notice?

2. Look at all the fractions *less* than $\frac{1}{2}$ and all the fractions equal to $\frac{1}{2}$. What do you notice?

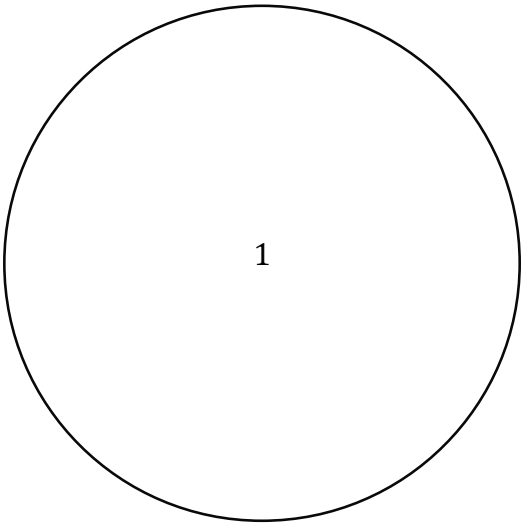
3. Look at all the fractions *more* than $\frac{1}{2}$ and all the fractions equal to $\frac{1}{2}$. What do you notice?

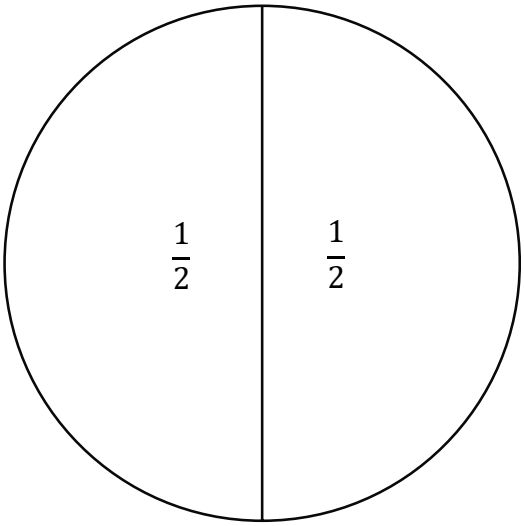
Fraction Sets

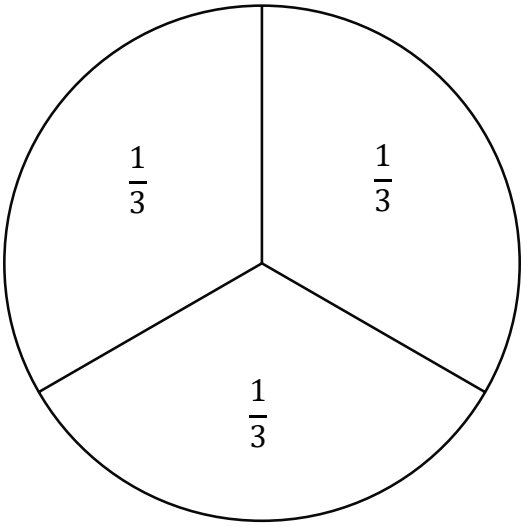
Ordering Fractions Cards

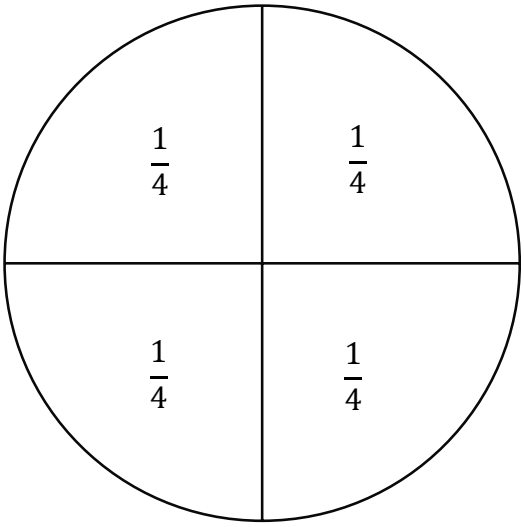
Set 1	Set 2
$\frac{11}{12}, \frac{1}{8}, \frac{3}{5}, \frac{1}{9}$	$\frac{1}{11}, \frac{4}{7}, \frac{3}{6}, \frac{4}{9}$
Set 3	**Set 4**
$\frac{5}{10}, \frac{2}{6}, \frac{3}{8}, \frac{6}{11}$	$\frac{9}{9}, \frac{2}{3}, \frac{3}{12}, \frac{10}{8}$
Set 5	**Set 6**
$\frac{1}{4}, \frac{7}{8}, \frac{4}{8}, \frac{9}{10}$	$\frac{5}{6}, 1\frac{1}{3}, \frac{9}{5}, \frac{8}{8}$

Fraction Circles (To Fourths)

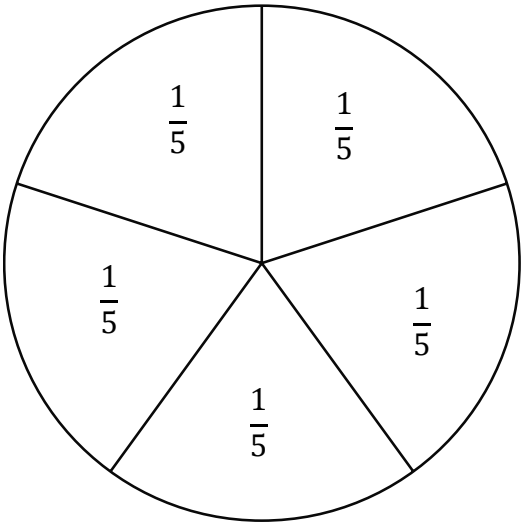


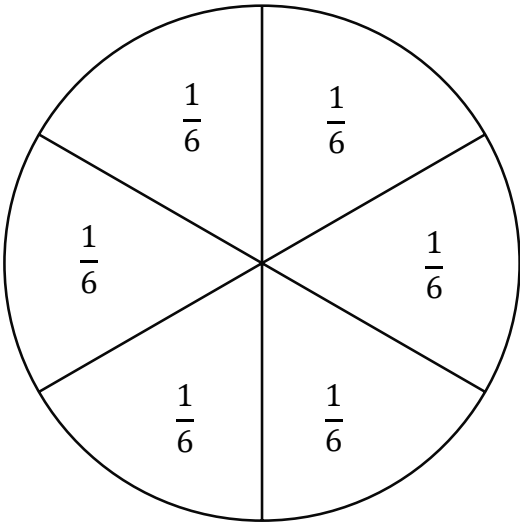


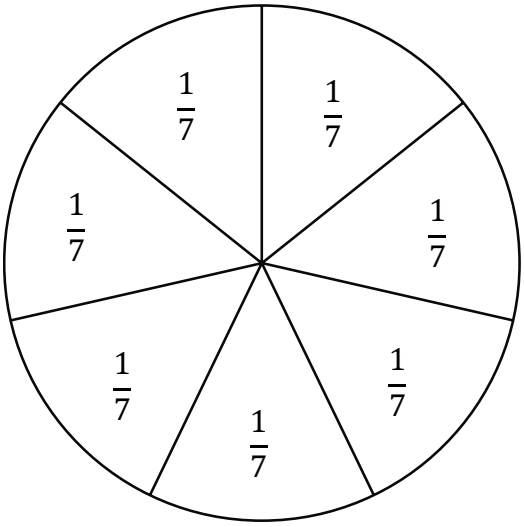


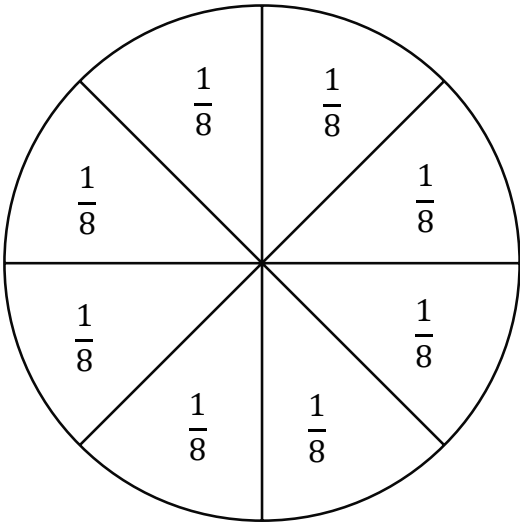


Fraction Circles (Fifths to Eighths)

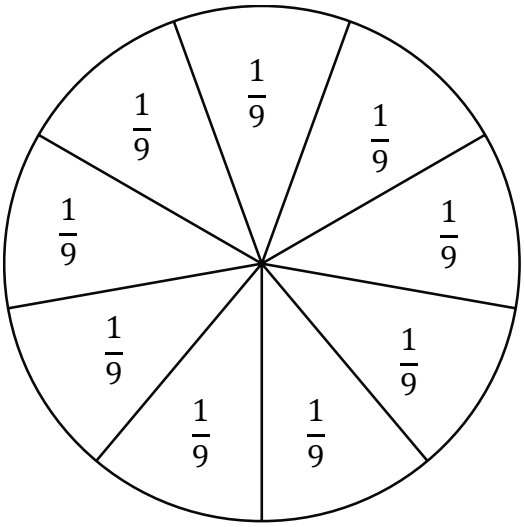


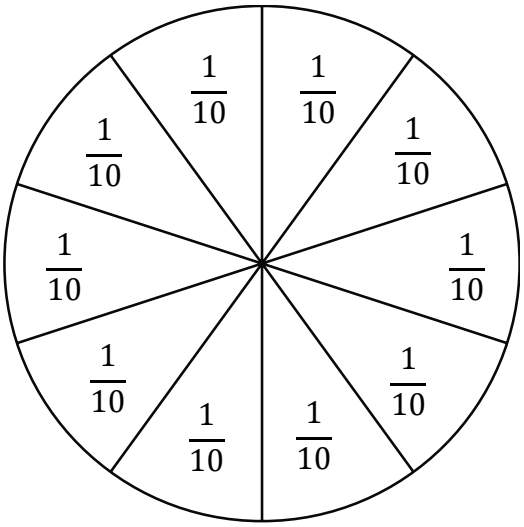


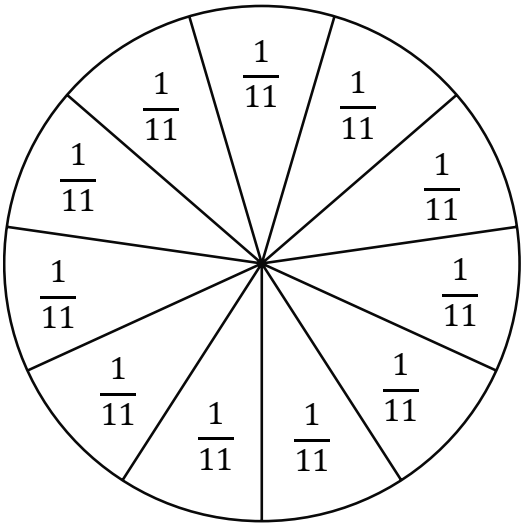


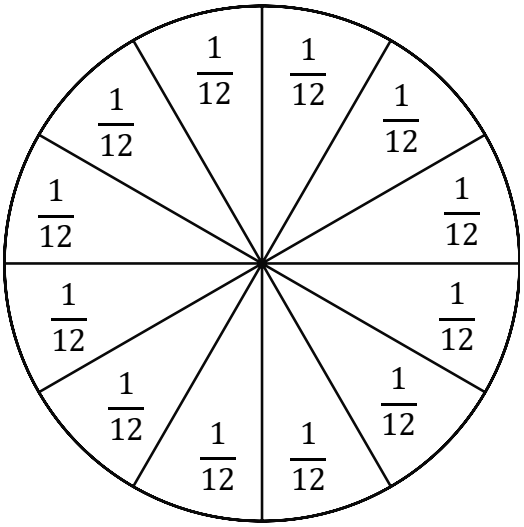


Fraction Circles (Ninths to Twelfths)









Fraction Sorting Mat

Less than ½	Equal to ½	Greater than ½

Fraction Cards

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{2}{4}$
$\frac{3}{4}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$
$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$
$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$
$\frac{6}{8}$	$\frac{7}{8}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$

$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$
$\frac{9}{10}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$
$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$
$\frac{10}{12}$	$\frac{11}{12}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$
$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$