Adding and Subtracting Fractions: Understanding the Context

Strand: Computation and Estimation

Topic: Adding and subtracting fractions using context as a tool to support student understanding

Primary SOL: 4.5 The student will
    c) solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.

Related SOL: 4.2b, 4.4d, 4.5a, 4.5b

Materials

- Pencil and paper
- Dry-erase boards and markers
- Manipulatives (including, but not limited to: fraction strips, number lines, fraction circles, Geoboards, and Geobands)
- Using Context to Make Sense of a Problem activity sheet (attached)
- Understanding the Context: Sample Problems activity sheet (attached)

Vocabulary

add, addition, computation, denominator, difference, estimation, fraction, improper fraction, greatest common factor, least common multiple, mixed number, numerator, subtract, subtraction, simplest form, sum

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Arrange students into pairs. Consider the learning preferences and knowledge level of your students to determine the best structure for sharing work, having meaningful discussions, and their documentation of thinking and problem solving.

2. Tell students that they will be presented with a problem and that partners should work together to read the problem carefully. Provide students with the option of using concrete manipulatives, pictorial representations, pencil and paper, or dry-erase boards and markers to communicate their thinking with each other. Write the following focus questions on the board: *"How can thinking about the context of a problem help you in understanding the situation and what the question is asking you to solve? How can this help you to determine a way to solve this problem?"* Have partners discuss the importance of the focus question and share their ideas with the class.

3. Distribute the Using Context to Make Sense of a Problem activity sheet. Write the problem on the board so that you can refer to the problem: "A school uses part of a field to create a rectangular area for the PTA carnival. They will partition the field so there are areas for games, food, and prizes. If $\frac{1}{6}$ of the field is for prizes, and $\frac{1}{2}$ of the field is for games, what fraction of the field is remaining for food?" Ask students to read the problem to themselves. Then call on a volunteer to read it out loud for the class.

4. Let students know they are not going to solve the problem right now, but you want them to think about how the context of the story problem can help them plan a way to solve the problem. Let them know that the context is like a story, and it can help them get a picture in their heads of what is happening in the problem. Point to the *focus question* on the board and ask the partners to think about and discuss the context of the problem and be able to describe how the context helps them understand the situation and the question they need to answer. Based on the context, what do they think they need to do to solve the problem? Partners should be allowed time to think and discuss the problem before a whole-class discussion.

5. Circulate around the room, listening to the group discussions. Take note of things you want to highlight or clarify during the whole-class discussion. It is important to encourage partners to think and reason about the situation in order to understand the context and be able to determine the solution. It is also important to encourage students to think about and discuss justifications on how the context of the problem leads to the actions required for a computation and what operation is associated with the computation. Some questions that could be used to push student discussions, if they are not sharing their thinking and reasoning, may include:
    a. What picture or little movie do you get in your head when you read about the situation?
    b. Could drawing a picture of the field be helpful to understanding the problem?
    c. What is the question about the situation that needs to be answered?
    d. What are some things to consider when figuring out what this situation's problem is asking you to find out?
    e. What operation or operations do you think the situation calls for when you get ready to solve the problem? Why?
    f. If you were going to use a manipulative to help you, which would you select and why?

6. Bring the class back together for a whole-class discussion. First, present students with a rephrasing or repeating of the first part of the focus question: *"How can thinking about the context of a problem help in understanding the situation and what the question wants you to find?"* Select a few partners to share their group's thoughts and the ideas that were discussed. Ask students to explain what in the context led them to their decisions. Highlight helpful suggestions, bring out points that students may not make that will also be helpful, and clarify any misconceptions or confusion that you noted while listening in on conversations.

7. Next, discuss the second part of the focus question: *"How can this help you to determine a way to solve this problem?"* Have some of the partners you previously identified share their ideas about the answer to the question. Ask students to provide justification from the context to support their decisions. Highlight helpful suggestions, bring out points that students may not make that will also be helpful, and clarify any misconceptions or confusion that you noted while listening in on conversations. Make explicit that the *action* in the problem can help decide what operation is called for to solve the problem. Have students share how the action in this situation calls for both addition and subtraction.

8. End discussions and present students with a rephrasing of the focus question and a request to now estimate the answer: *"Now, we have talked about how understanding the problem by thinking about its context can help you to determine a way to solve this problem. So, the next step is to come up with a reasonable estimate for the answer to the question. Talk with your partner for a couple of minutes and between the two of you estimate what you think the answer will be and write the estimate on your paper."* As a formative assessment and to provide information about where individual students are in their understanding, debrief the estimates by making true/false statements, such as those below, and asking students to use thumbs-up if they agree and thumbs-down if they do not agree.

    a. The answer is less than 1.
    b. The answer is more than $\frac{1}{2}$.
    c. The answer is less than $\frac{1}{2}$.
    d. The answer is more than 0.
    e. The answer is closer to 0 than $\frac{1}{2}$.

9. Ask pairs to work together to find an exact answer to the problem, using manipulatives, pictures, and words to explain their answer. Ask students to write a number sentence to show how to solve the problem and the final solution. Remind them that the answer to a contextual problem is always written as a sentence, such as: The fraction of the field that can be used for food setup is _____. Ask the students whether there is more than one way to solve this problem.

10. Circulate around the room and make note of how students are solving the problem, and which students you want to share their work using a demonstration tool (e.g., document camera, digital display) or writing it on the board. Let the students know that you will be asking them to share and explain their work. Also, note how much attention you will need to give to reviewing finding common denominators and equivalent fractions. Facilitate a class discussion about the process used for solving the problem with a manipulative, a pictorial representation, and a number sentence. Help students make the connections about the mathematics involved as shown by each model.

11. Use one or both of the Understanding the Context: Sample Problems for student pairs to engage in additional practice. The problems on page 1 include fractions where one of the denominators is the least common multiple. On page 2, none of the denominators of the fractions in a problem is the least common multiple.

Assessment

- **Questions**
    - How can understanding a real-world problem, through thinking about its context, help you to determine a way to understand the situation and to solve the problem?
    - What are some strategies you can use to help you understand the context of a real-world problem before solving the problem?

- o In what ways can you represent your understanding about the situation in a real-world problem? Share an example of representing your understanding of a problem you solved recently.

- **Journal/writing prompts**
    - o Use pictures, symbols, and words to show and discuss your understanding of the context of the following real-world problem: Mom baked a cake. She gave $\frac{1}{6}$ of the cake to a neighbor and served $\frac{3}{4}$ of the cake to friends for lunch. Was there any cake left over? If so, how much?
    - o Write a real-world problem about a recipe where two fractional parts have to be combined.
    - o Write a real-world problem comparing two fractional parts to find out how much greater one is than the other.

- **Other Assessments**
    - o After a student demonstrates success with an initial way to understand a given problem, ask, *"Are there any other ways to demonstrate your understanding of this real-world problem?"*
    - o Analyze students while they work in pairs or while working independently. Do students show they understand the problem? How do they show it? Why do they understand? Why do they not understand?
    - o Pick a student work sample from one or more of the Understanding the Context: Sample Problems and remove the student's name. Present students with a copy of the work and have them analyze the work sample and respond to a focus question, such as: Did the student show understanding of the situation in the problem? Cite evidence to justify why or why not. Did the student find the correct answer? If not, identify the student's misconceptions or errors made when solving the problem.

Extensions and Connections

- Write a real-world problem that can be represented by the equation: $? - \frac{1}{4} = \frac{5}{8}$.
- Have students find a real-world example where fractions must be added or subtracted.
- Conduct a science experiment where fractional amounts of ingredients to make something must be added or subtracted.

Strategies for Differentiation

- Use fractions with like denominators, in substitution for fractions with unlike denominators in the problem, to make the context and computation easier to think about.
- Use simple whole numbers, in substitution for the fractions in the problem, to make the context and computation easier to think about.

- Some students may understand a problem by drawing a picture and then using that picture to help them first solve it without a computation. The teacher can then use their picture to help them bridge from the pictorial representation to the symbolic.
- Some students may be more successful understanding the context if allowed to act out the scenario with manipulatives or other students.
- Allow students who can successfully show understanding of real-world problems involving fractions to write their own real-world problems for other classmates to solve.
- Given only a drawing of a solution to a problem, have students write a real-world problem that may have been presented to the student who drew the solution.
- Provide the students with additional manipulatives to use when solving the problem.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

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Using Context to Make Sense of a Problem

A school uses part of a field to create a rectangular area for the PTA carnival. They will partition the field so there are areas for games, food, and prizes. If $\frac{1}{6}$ of the field is for prizes, and $\frac{1}{2}$ of the field is for games, what fraction of the field is remaining for food setup?

Understanding the Context: Sample Problems (Page 1)

Directions: Read each problem carefully and think about the context of the problem: What is the situation, and what is the question? Provide a reasonable estimate and a justification for the estimate based on the context of the problem. Consider using an appropriate manipulative to estimate and then find an exact solution to the problem. Represent your solution in a drawing and with an equation. All solutions that include fractions should be in simplest form.

A woman walks $\frac{1}{4}$ of a mile and stops to do some push-ups. After the push-ups, she continues walking and notices that she has walked a total of $1\frac{1}{2}$ mile. What was the distance the woman walked after doing her push-ups?	Marissa bought $1\frac{3}{8}$ yards of pink cotton fabric and $1\frac{3}{4}$ yards of white silk fabric. How much fabric did she buy?	Mr. Hill had some rope in his garage. He used $1\frac{5}{6}$ meters of the rope to hang a small swing. His wife used the remaining $\frac{1}{3}$ of a meter from the same rope for a craft project. How many meters of rope were there to begin with?

Understanding the Context: Sample Problems (Page 2)

Directions: Read each problem carefully and think about the context of the problem: What is the situation, and what is the question? Provide a reasonable estimate and a justification for the estimate based on the context of the problem. Consider using an appropriate manipulative to estimate and then find an exact solution to the problem. Represent your solution in a drawing and with an equation. All solutions that include fractions should be in simplest form.

Heath purchased $\frac{3}{4}$ of a pound of almonds and $1\frac{1}{3}$ pound of peanuts. How many pounds of nuts did he buy?	Two friends shared a pack of 12 colored pencils. Katrina used $\frac{1}{4}$ of the pack and Lester used $\frac{2}{3}$ of the pack. What fraction of the pack of pencils were not used?	Which of the following pieces of fruit weighs less? How much less? • An apple weighing $\frac{1}{2}$ of a pound • A banana weighing $\frac{2}{3}$ of a pound