Order Up! Equivalences and Ordering Fractions and Decimals

Strand:	Number and Number Sense
Topic:	Recognizing equivalent relationships and comparing and ordering fractions and decimals.
Primary SOL:	5.2 The student will

        a) represent and identify equivalencies among fractions and decimals, with and without models;* and

        b) compare and order fractions, mixed numbers and/or decimals, in a given set, from least to greatest and greatest to least. *

        * On the state assessment, items measuring this objective are assessed without the use of a calculator.

Materials
- Base-10 blocks
- Decimal Grid Paper (attached)
- Decimal Strips Paper (Tenths) (attached)
- Decimal Grid (Hundredths) (attached)
- Scissors
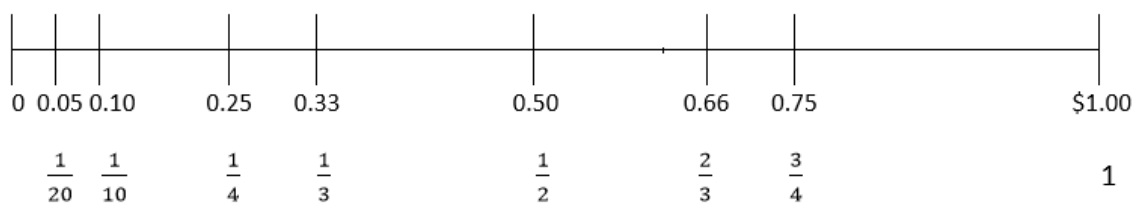- Pen or marker

Vocabulary
decimal, denominator, equal (=), equivalent, fraction, greater than (>), greatest, hundredth, improper, least, less than (>), mixed number, not equal (≠), numerator, proper, repeating decimals, tenth, terminating decimals, whole

Student/Teacher Actions: What should students be doing? What should teachers be doing?
Note: It is important that students understand commonly used fraction/decimal equivalents because of their use in various real-world contexts. This lesson will review the familiar fraction/decimal equivalents using money and number lines, and then use this knowledge to develop strategies to find the equivalences for less familiar fractions.

1. Ask, *"Do you know any equivalent decimals for fractions? Hint: Think about fractions of a dollar. Discuss with your elbow partner."* Allow a few minutes for partners to discuss, and then ask volunteers to share. As volunteers share ideas, draw a number line on the board, from 0 to $1.00, and add students' suggestions to the number line. Fractions for 0.05, 0.10, 0.25, 0.50, and 0.75 are easiest for students to remember due to their knowledge of coin values. (See the number line below for fractions and their equivalent decimals that students may recall. Do not push for more responses at this time than students are easily able to recall. Thirds and eighths are more challenging, and students need time to investigate with these fractions. The activity that follows will help all students to develop the other equivalences in a meaningful way.)

   The final number line might look like this:

0	0.05	0.10	0.25	0.33	0.50	0.66	0.75			$1.00
	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$			1

When working with halves, fourths, fifths, and tenths, encourage students to continue thinking in terms of fractions of a dollar and the equivalent cents when comparing and ordering fractions and decimals.

Discovering patterns: Create an anchor chart like the one below to explore patterns among the decimal and fraction relationships. Start with the column headings and blank cells in the table. Fill in the cells systematically during the class discussion so that students will more likely be able to identify the relationships by examining the patterns.

Fraction	Decimal		Fraction	Decimal		Fraction	Decimal
						$\frac{1}{8}$	
			$\frac{1}{4}$			$\frac{2}{8}$	
						$\frac{3}{8}$	
$\frac{1}{2}$			$\frac{2}{4}$			$\frac{4}{8}$	
						$\frac{5}{8}$	
			$\frac{3}{4}$			$\frac{6}{8}$	
						$\frac{7}{8}$	0.875
$\frac{2}{2}$	1.00 or 1		$\frac{4}{4}$	1		$\frac{8}{8}$	1

- Ask students to explain how they know one-half is equivalent to 0. 50 and 0.5—*"How do you know that we can write either 0.50 or 0.5?"* Record the equivalence in the table. Ask, *"What decimal is $\frac{2}{2}$ equivalent to? Why do you think $\frac{2}{2}$ = 1?"* Record the equivalence in the table and continue doing this during the remainder of the discussions as further equivalences are established.

- Ask, *"What decimal is equivalent to $\frac{1}{4}$, and how do you know? What decimal is equivalent to $\frac{2}{4}$, and how do you know? $\frac{3}{4}$? $\frac{4}{4}$?"*
  - Have students work alone and then use think-pair-share to investigate the patterns in the columns and rows of the chart.
  - Have students discuss their ideas with a partner.

- Facilitate a whole-class discussion for students to share their thoughts and discoveries.
- Use the students' ideas to highlight relationships among the columns.
    - For example, a student might state, "I know that $\frac{2}{4}$ = 0.50, and because $\frac{1}{4}$ is half of $\frac{2}{4}$, I know the decimal for $\frac{1}{4}$ is half of 0.50, or 0.25. And $\frac{3}{4}$ is $\frac{1}{4}$ more than $\frac{2}{4}$, so the decimal for $\frac{3}{4}$ is 0.50 added to 0.25, or 0.75."
    - Also, give students a chance to discover the relationships across the rows, such as $\frac{1}{2} = \frac{2}{4} = 0.50$.

This is what the table should like at the end of the discussions.

Fraction	Decimal		Fraction	Decimal		Fraction	Decimal
						$\frac{1}{8}$	0.125
			$\frac{1}{4}$	0.25		$\frac{2}{8}$	0.25
						$\frac{3}{8}$	0.375
$\frac{1}{2}$	0.50 or 0.5		$\frac{2}{4}$	0.50		$\frac{4}{8}$	0.50
						$\frac{5}{8}$	0.625
			$\frac{3}{4}$	0.75		$\frac{6}{8}$	0.75
						$\frac{7}{8}$	0.875
$\frac{2}{2}$	1.00 or 1		$\frac{4}{4}$	1		$\frac{8}{8}$	1

2. Once students discuss halves and fourths, examine their relationships among the fraction and decimal equivalence, and use them to figure out the ones they do not recall, they are ready to work with the eighths.

3. First, ask students how they know $\frac{4}{8}$ = 0.50. Next, ask students about $\frac{2}{8}$ = 0.25. How could they figure this out two different ways? Some students will be able to explain that $\frac{2}{8} = \frac{1}{4}$, leading to the fact that both must equal 0.25. Ask students to use two different ways to explain the equivalent decimal for $\frac{6}{8}$. Then ask students to work with their partners to figure out the decimal equivalents for the remaining eighths in the chart and to be prepared to explain how they used what they knew to figure out the ones they did not know. Circulate around the room and note the students who are able to use the relationships and those who cannot use them correctly. When most of the class is

finished, facilitate a class discussion to complete the chart, asking students to explain their thought processes.

4. Provide base-10 blocks or a 10-by-10 decimal grid (which represents pennies in a dollar) for student to use as they see whether they can figure out/show how much money is equivalent to $\frac{1}{3}$. Use the relationship between decimals and money to help students begin to think about the decimal and fraction equivalents for $\frac{1}{3}$ and $\frac{2}{3}$. Using the base-10 blocks or 10-x-10 grid, ask students to divide 100 into three equal groups. Ask, *"How would you write a money value for$\frac{1}{3}$?"* Circulate around the room to see what strategies students are using. Discuss students' answers. You may have identified students whose strategies you would like to share.

5. Use the students' responses to talk about how the grid with 100 little squares could be divided/partitioned into three equal parts and how each of those parts had 33 little squares but there was one square left over. Some students may be able to discuss that the one little square could be divided into thirds. Lead the discussion to where students can see that 1.00 divided by 3 is .33$\frac{1}{3}$, but there is still a fraction. Explain that $\frac{1}{3}$ is another 33 tiny squares with one left over and that it keeps going. Because the 3 keeps repeating, the convention is to write this as 0.3333... . Mathematicians write this as (.3$\overline{3}$). Decimals the repeat in a pattern and do not terminate or end are called repeating decimals.

6. Next, ask students to figure out the equivalent decimal for $\frac{2}{3}$. Some may realize they can use the relationship that $\frac{2}{3}$ is twice as much as $\frac{1}{3}$, and twice 0.333 is 0.666, which can also be represented as 0.6$\overline{6}$. Include these relationships on the anchor chart.

Ordering Fractions and Decimals

7. Present students with the following fractions and decimals: 0.3, $\frac{1}{8}$, .83, and $\frac{2}{5}$. Group students in pairs or threes. Challenge students to discuss how to order these from least to greatest. Encourage them to use strategies they have used today: number line, money, fraction, and decimal benchmarks, anchor chart, and place value (0.3 is equivalent to 0.30 and 0.300). Ask students to record their thinking and to be prepared to explain how they decided on the order. Circulate around the room and note what strategies students are using and which students seem to have misconceptions or confusion.

8. Call on the groups you identified to share and explain their thinking. Important questions to ask:
   - "How did your group order these fractions and decimals?
   - Why did you order them that way? Does anyone have them in a different order?
   - Why did you decide 0.3 is greater than $\frac{1}{8}$?
   - How did you decide where to place 0.83? Do other groups agree?
   - Did anyone figure out the decimal equivalent for $\frac{2}{5}$? How did you figure it out? Did anyone think about this differently?

9. Encourage students to describe and defend their reasons for placing fractions and decimals in particular spots in the list, including if they used number lines, benchmarks, equivalent fractions, changing decimals to fractions or fractions to decimals, paper folding, or base-10 blocks.

10. Guide students to agree on the following order from least to greatest:

$$\frac{1}{8}, 0.3, \frac{2}{5}, 0.83$$

11. As an exit ticket, have students use pictures, words, and symbols to describe how you would list 1.5, $\frac{1}{5}$. and 0.15 in order from least to greatest.

Assessment

- **Questions**
  - Create a chart that shows the fraction and decimal equivalents for tenths -- $\frac{1}{10}$, $\frac{2}{10}, \frac{3}{10}, \frac{10}{10}$ —and describe how you determined the decimal equivalences.
  - Which decimal is larger: 0.125 or 0.2? Show how you know with pictures, symbols, and words.
  - Which is smaller: 0.875 or $\frac{7}{8}$? Show how you know with pictures, symbols, and words.
  - Give the students a number line and have the students order fractions and decimals from least to greatest. The number line can be created out a sentence strip or drawn on a sheet of paper. A suggestions of decimals and fractions are included below that students can order from least to greatest.
    - $\frac{3}{2}$, 0.9, 0.51, $1\frac{1}{5}$
    - $\frac{3}{10}$, 0.37, 0.71, $\frac{3}{5}$
    - $\frac{2}{3}$, 0.57, $\frac{6}{7}$, 0.43
    - $\frac{10}{12}$, 0.6, $1\frac{1}{6}$, 0.42
    - 0.431, $\frac{5}{9}$, $\frac{6}{12}$, 0.97

- **Journal/writing prompts**
  - Use pictures, symbols, and words to explain how you would compare $\frac{5}{8}$ and 0.75.
  - Explain, using pictures, symbols, and words, how you would order 1.5, $1\frac{3}{5}$, 1.45, and $1\frac{3}{8}$ from greatest to least.

- **Other Assessments**
  - Ask students to list all of the fraction and decimal equivalences they know, using change amounts less than $1.00.
  - Ask students to explain the difference between repeating and terminating decimals.

Extensions and Connections

- Tape a decimal or fraction to the back of each student. Then have them ask the other students yes-or-no questions to try to figure out their number.

- Have students fold pieces of paper into four parts. Have them write the fraction in one part, write the equivalent decimal in another part, draw a picture of the fraction in the third part, and write a connection to the real world in the fourth part (e.g., $\frac{1}{4}$ can be represented by a coin, a quarter).

- Students can play a game similar to "Go Fish" with fraction decimal equivalences. The fractions included are represented in simplest form. To play the game, the students will deal each player 5 cards and place the remaining cards in the middle of the playing area to be used as the "Go Fish" pile. Players take turns asking another player for a card. If the player is holding a decimal card, then he/she will ask another player for the equivalent fraction in simplest form. For example: if you have the decimal card 0.22 the player should ask another player for the fraction $\frac{11}{50}$, instead of $\frac{22}{100}$. If the player has the card, he/she must give it to the player who asked for it. The player then lays down the matched pair. If the other player does not have the desired card, then the player should respond with "Go Fish" and player 1 should pick another card from the pile. The first player to match all of their card, and thus have no card remaining, is the winner.

- Have students work together in a small group or with a partner ordering fractions and decimals from least to greatest by placing the cards on a clothesline. For this activity 5 different deck of cards have been created for students to order. Students work together to order the fractions and decimals by placing them on a "clothesline" created out of yarn or string. After the students have organized the fractions and decimals on the clothesline they can record the information on the recording sheet "Fraction and Decimals…Out to Dry" (attached). Once the students have completed one deck, the students can rotate to the next deck of cards until all 5 decks have been ordered.

Strategies for Differentiation

- Challenge students to find other denominators that would be repeating. Do they have anything in common?

- Explore equivalent fractions and decimals with Decimal Strips Paper or base-10 blocks:
  - Using the Decimal Strips Paper activity sheet, give each student at least seven 1" × 10" strips with each strip divided into 10 equal parts. Ask students to examine the grid side of the strip and determine the number of parts in the whole (10). Ask, *"If you wanted to name the parts of the strip, what name would you give each part?"* The answer is "one-tenth." Discuss the two ways to represent tenths—fraction and decimal.
  - Have each student fold a strip in half and cut it. Using the blank side of the strip, discuss the fraction name for each part and label each accordingly ($\frac{1}{2}$). Have each student turn the strip over and discuss the decimal name for each part by counting the tenths (five tenths). Have students label each part (0.5) and set those pieces aside.

- o Have each student fold another strip in half two times and unfold it. Discuss the fraction name for the parts (four parts in the whole, each part is one fourth). Have each student make one cut on the first fold. Ask students what part of the whole the small piece represents ($\frac{1}{4}$) and what part the larger piece represents ($\frac{3}{4}$). Have students label the blank sides of those pieces with the appropriate fraction. Direct students to turn the strip over to examine the grid side. Ask students how many tenths they see when looking at the smaller piece (halfway between two tenths and three tenths). Ask what number is exactly halfway between two tenths and three tenths (.25); use a money connection, if necessary *("What is halfway between two dimes and three dimes?")*. Have students label the grid sides of those pieces with a decimal number. Repeat the process with the larger piece (.75).

- o Using other strips, have students make folds to represent eighths, thirds, fifths, and tenths and repeat the labeling process. Each time, have each student cut one unit fraction off the whole and leave the rest of the strip intact. Have students label the blank side of each piece with the appropriate fraction and the grid side with a decimal.

- o When six strips have been folded, cut, and labeled, have each student work with a partner to put all 12 of their pieces in order from smallest to largest. One partner should use the fraction side of their pieces, while the other uses the decimal side.

- o Alternating between using fraction and decimal names, call on individual students to form a line at the front of the room holding their strip pieces in order from smallest to largest. *("Will someone bring the smallest piece to the front of the room with the fraction side showing? Will someone else bring the next smallest piece with the decimal side showing and line up right beside the first person?")*

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Decimal Strips Paper (Tenths)

Decimal Grid (Hundredths)
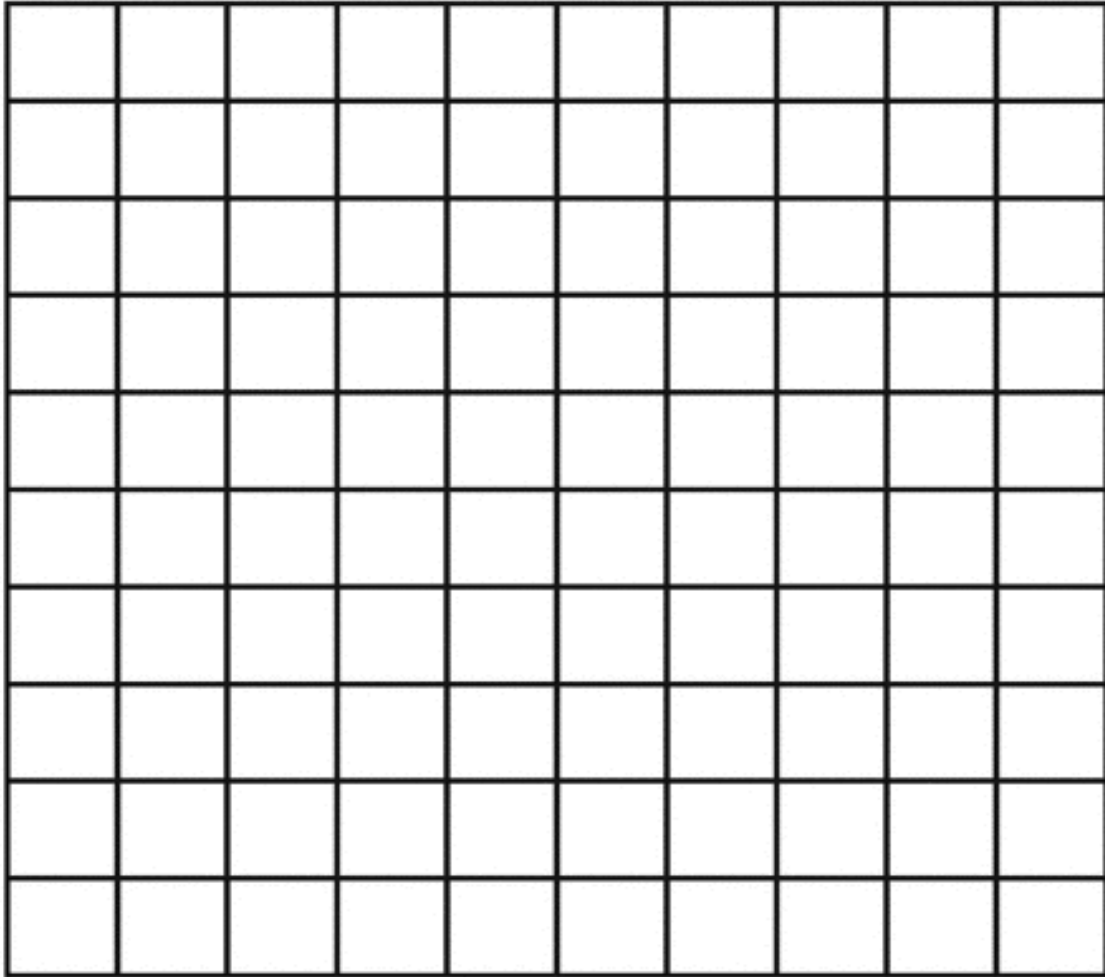
Fishing for Decimals and Fractions
Game Cards

$\dfrac{1}{2}$	0.5	$\dfrac{1}{5}$	0.2	$\dfrac{1}{4}$
0.25	$\dfrac{3}{5}$	0.6	$\dfrac{3}{4}$	0.75
$\dfrac{3}{8}$	0.375	$\dfrac{3}{10}$	0.3	$\dfrac{4}{5}$

0.8	$\dfrac{1}{8}$	**0.125**	$\dfrac{2}{5}$	**0.4**
$\dfrac{5}{8}$	**0.625**	$\dfrac{7}{8}$	**0.875**	$\dfrac{7}{10}$
0.7	$\dfrac{9}{10}$	**0.9**	$\dfrac{1}{3}$	**0.$\overline{3}$**

$\dfrac{2}{3}$	$0.\overline{6}$	$\dfrac{63}{100}$	0.63	$\dfrac{41}{50}$
0.82	$\dfrac{49}{50}$	0.98	$\dfrac{17}{20}$	0.85
$\dfrac{24}{25}$	0.95	$\dfrac{7}{20}$	0.35	$\dfrac{1}{25}$

0.04	$\dfrac{23}{50}$	0.46	$\dfrac{23}{100}$	0.23

Fishing for Decimals and Fractions

Number of players: 2 to 4 players

Materials:

- Decimal fraction cards
- Scratch paper

Directions:

- Deal 5 cards to each player and place the remaining cards in the middle of the playing area to be used as the "Go Fish" pile.
- Players take turns asking another player for a card. If the player is holding a decimal card, then he/she will ask another player for the equivalent fraction in simplest form. For example: if you have the decimal card $\boxed{0.22}$ the player should ask another player for the fraction $\frac{11}{50}$, instead of $\frac{22}{100}$.
- If the player has the card, he/she must give it to the player who asked for it. The player then lays down the matched pair. If the other player does not have the desired card, then the player should respond with "Go Fish" and player 1 should pick another card from the pile.
- The first player to match all of their card, and thus have no card remaining, is the winner.

Fractions and Decimals...Out to Dry

Name _____   **Date**_____

DECK A
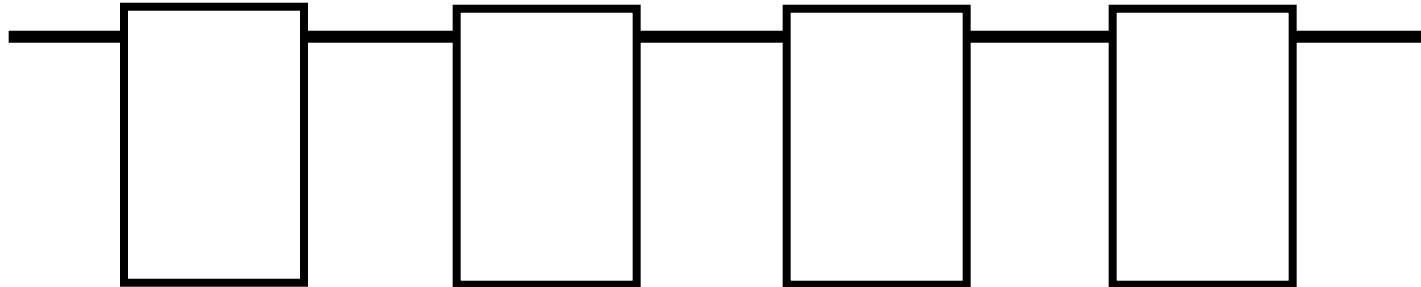
DECK B
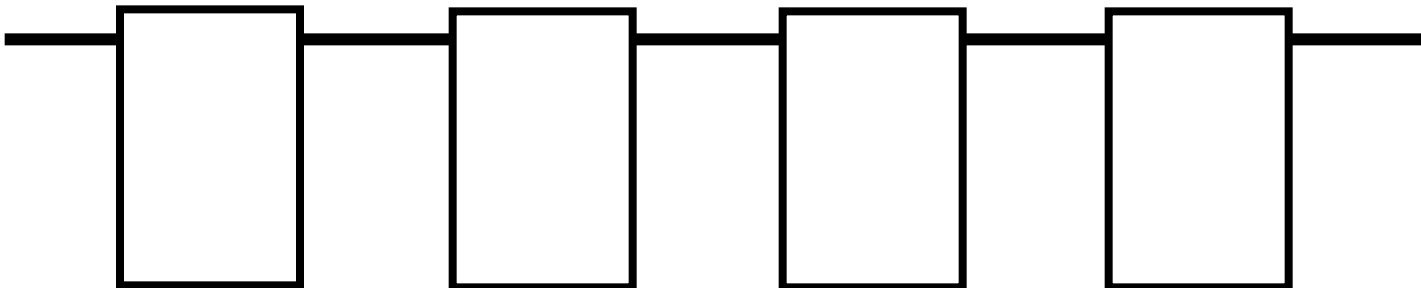
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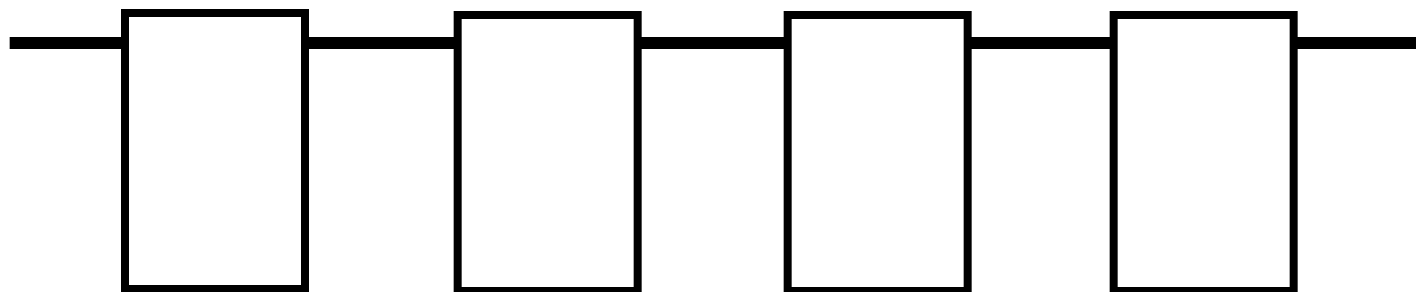
DECK D
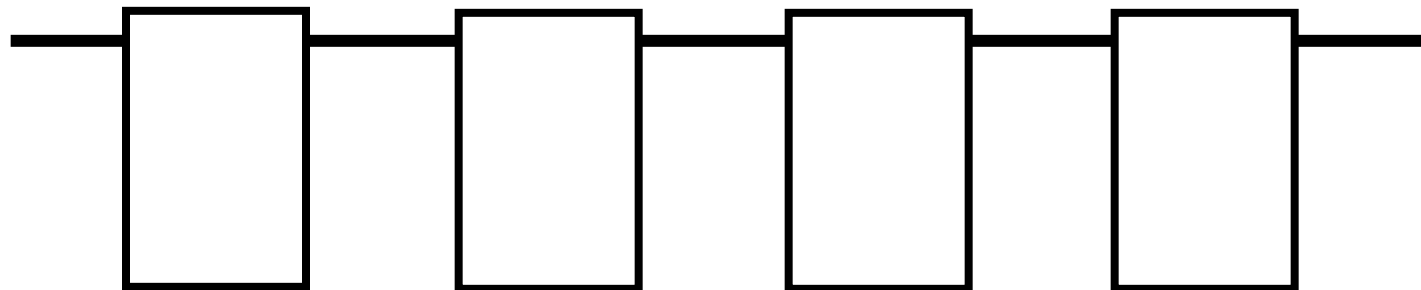
DECK E
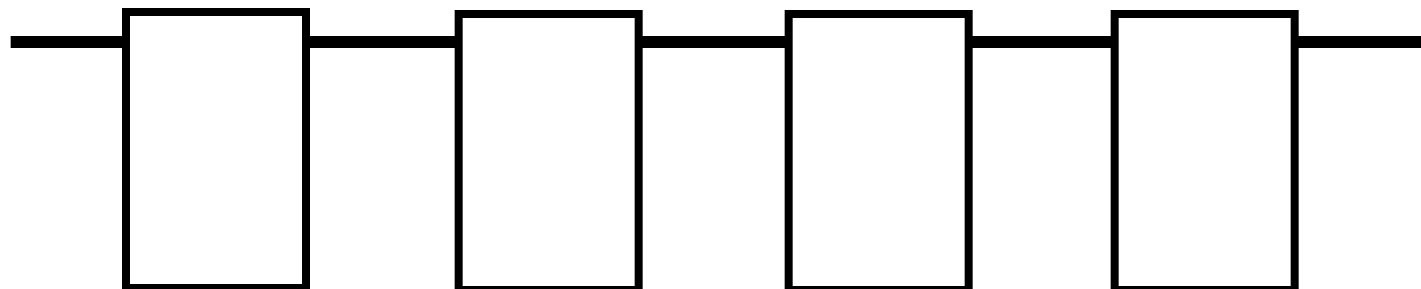
Deck Cards

Deck A:

0.625	$\dfrac{3}{4}$	$1\dfrac{1}{3}$	0.15

Deck B:

$\dfrac{11}{12}$	$\dfrac{6}{9}$	0.59	$\dfrac{4}{3}$

Deck C:

$\dfrac{9}{5}$	1.5	0.96	$\dfrac{6}{8}$

Deck: D

$\dfrac{5}{11}$	$\dfrac{8}{12}$	0.567	$\dfrac{9}{10}$

Deck: E

2.67	$2\dfrac{1}{3}$	3.2	$\dfrac{9}{3}$