

Rich Mathematical Task – Grade 5 – *Bus A or Bus B?*

Task Overview/Description/Purpose:	
	<ul style="list-style-type: none"> The purpose of this task is for students to explore strategies (counting up, number line, t-chart, etc.) in order to develop and deepen their mathematical understanding of elapsed time. Students will create a strategy for determining elapsed time in order to determine which of two bus trips is the better deal (according to their own criteria). Prior to this task, students will have had experiences determining elapsed time to the hour and minute within a 12-hour period.

Standards Alignment: Strand - <i>Measurement</i>	
Primary SOL:	SOL 5.11 The student will solve practical problems related to elapsed time in hours and minutes within a 24-hour period.
Related SOL (within or across grade levels/courses):	3.9b, 4.9

Learning Intentions:	
	<ul style="list-style-type: none"> Content - I am learning how to apply a strategy for determining elapsed time within a 24-hour period. Language - I am learning to describe how to compare and contrast two elapsed time situations. Social - I am learning to receive and value feedback about my mathematical thinking.

Evidence of Student Learning (based on Essential Knowledge and Skills):	
	<ul style="list-style-type: none"> I can solve practical problems related to elapsed time in hours and minutes within a 24-hour period: when given the beginning time and the ending time, determine the time that has elapsed; I can communicate my mathematical thinking to my peers using comparative language about elapsed time. I can use at least one representation to support my mathematical thinking. I can make connections between different representations of mathematical thinking.

Mathematics Process Goals	
Problem Solving	<ul style="list-style-type: none"> Students will choose an appropriate and efficient problem solving strategy to determine elapsed time. Students will accurately apply the strategy to obtain a valid solution.
Communication and Reasoning	<ul style="list-style-type: none"> Students will communicate their thinking process of the best choice of bus by determining the total time for both buses by using comparative language. Students will demonstrate sound reasoning and justify solution steps in an organized and coherent manner. Students will use appropriate mathematical language to express ideas, including elapsed hours and minutes, am and pm, with accuracy and precision.
Connections and Representations	<ul style="list-style-type: none"> Students will use at least one clear and appropriate representation to explore and model the problem. Students will describe connections between their representation and the representations of their peers. Students will make connections and/or extensions to other mathematical ideas.

Task Pre-Planning	
Approximate Length/Time Frame: 60 minutes	

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Grouping of Students: Students will begin working on the task independently for approximately 5 minutes. Then students will be partnered in groups of 2-3 students based on teacher monitoring and recording of strategies on checklist.

Materials and Technology:

- task copies
- paper, pencils
- student clocks
- bus chart supporting document
- blank number lines supporting document

Vocabulary:

- elapsed time
- hours, minutes
- am and pm
- comparative language: longer, shorter, more/less/fewer

Anticipate Responses: See the Planning for Mathematical Discourse Chart (columns 1-3).

Task Implementation (Before) 5-10 minutes

Task Launch:

- **Activate prior knowledge:** Access students' prior knowledge of elapsed time by doing a short "Notice and Wonder" activity. Project the bus schedule found in the supporting documents or use the [Elapsed Time Flipchart](#). Ask students what they notice and wonder about the bus schedule. Give students a minute to observe. Then have students "Turn and Talk" to discuss what they notice. Record what students notice and wonder on chart paper or on the board. Possible responses: The bus ride to Virginia Beach is shorter than the trip to Washington, DC. The bus ride to New York arrives in the morning. If students do not mention it, ask students what is meant by the amount of time that has **elapsed** and what the difference is between am and pm. Do not ask them about strategies. We do not want to lead students to any one specific strategy for determining an elapsed time.
- **Check for student understanding of the task:** Allow students to read task. Then read the task to students. Ask students to "Turn and Talk" to restate what the task is asking without giving any answers. Possible student response: "We need to figure out which Bus Trip, Bus A or Bus B would be better and tell why." Have students read the "I Can Statements" so that they are aware of the expectations in addition to solving the task.
- **Establish clear expectations:**
 - Directions:
 1. For about 5 minutes I will give you time to think about and work on this task independently.
 2. While you are working I will walk around and make a note of the strategies you are using and I may ask you a question.
 3. Then I will ask you to stop and work with a partner or partners based on the work I see you doing. I encourage you to discuss your strategies and ask your partners questions so that you can better share your work with the class.
 4. Ask students if they have questions.

Task Implementation (During) 20-30 minutes

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see Question Matrix attached)
- Select – Teacher will decide which strategies or thinking will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation)
- Connect – Teacher will consider ways to facilitate connections between different student responses

Suggestions For Additional Student Support

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- Students who need more support using academic language for justifications could benefit from sentence frames such as:
 - The strategy I will use to determine elapsed time is _____.
 - I agree/disagree with _____'s strategy because _____.
 - I know the elapsed time is _____ because _____.
 - Bus ___ is the better option since it has less elapsed time. The difference is _____.
 - To compare the elapsed time for Bus A to Bus B, I
- Students with weaknesses in memory and/or vocabulary could benefit from:
 - Anchor chart to reference elapsed time strategies such as a number line or t-chart (charts could include various strategies for determining and comparing elapsed time)
- Students showing difficulty with visual-spatial skills may benefit from:
 - blank number lines as an organizer for students who have trouble transferring their thinking in an organized way or need a prompt to get started; or
 - analog clocks to use as a concrete model.

Task Implementation (After) 20 minutes

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion. Some possible mathematical ideas to highlight could include:
 - a common misconception;
 - trajectory in sophistication in student ideas (i.e. concrete to abstract); or
 - several different solutions for the same task.
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
 - How are these strategies alike? How are they different?
 - How is _____'s strategy like _____'s strategy?
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
 - Students can do a Gallery Walk to see all strategies after you highlight specific strategies.

Teacher Reflection About Student Learning

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- How will student understanding of the content be assessed:
 - Rich Mathematical Task Rubric
 - Self-Assessment through I Can Statements
- How will the evidence provided through student work inform further instruction?
 - Teacher will use the chart with anticipated student solutions to monitor which students are using which strategies. This will include: possible misconceptions, learning trajectories and sophistication of student ideas, and multiple solution pathways. Next steps based on this information could include:
 - Informing sequence of tasks. What will come next in instruction to further student thinking in elapsed time?
 - Informing small groups based on misconceptions that are not addressed in sharing.
 - Informing small groups based on movement along the learning trajectory/growing in sophistication of ideas (i.e. concrete to abstract)
- After task implementation, the teacher will use the Process Goals rubric to assess students' strengths and areas for growth in process goals. This could be a focus on one category. Next steps based on this information could include:
 - Informing sequence of tasks. What will come next in instruction to further student engagement in the process goal(s)?
 - Informing small groups based on where students are in engagement in the process goal(s) (i.e. sentence frames for communication, graphic organizers, strategic grouping, manipulative support, etc.).

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Planning for Mathematical Discourse

Mathematical Task: Bus A or Bus B?

Content Standard(s): SOL 5.11

Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
Anticipated Student Response/Strategy	Assessing Questions – Teacher Stays to Hear Response	Advancing Questions – Teacher Poses Question and Walks Away	List of Students Providing Response	Discussion Order - sequencing student responses
<p><i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i></p>	<p><i>Teacher questioning that allows student to explain and clarify thinking</i></p>	<p><i>Teacher questioning that moves thinking forward</i></p>	<p><i>Who? Which students used this strategy?</i></p>	<ul style="list-style-type: none"> ● <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> ● <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
<p>Anticipated Student Response: *Common Misconception Student will add or subtract the start time from end time and gets 67 minutes or 67 hours or some other incorrect amount. They may disregard or not understand the difference between am and pm.</p> <p style="text-align: center;">8:14pm <u>7:47am</u> 67hrs or 67 min.</p>	<ul style="list-style-type: none"> ● Tell me about your thinking. ● Can you tell me what am and pm mean? ● How many hours are in one day? ● How many minutes are in one hour? ● Does that answer sound like a reasonable amount to time to spend on a bus trip? ● Can you use the clock to show me what 1 hour from the start time would be? two hours? 	<ul style="list-style-type: none"> ● Could your partner write down your process as you say it? ● Could you use a drawing or representation to prove your thinking? ● Is there another method or model that you could use to show the elapsed time? 	See Student C	
<p>Anticipated Student Response: Student counts up time in their head or to themselves and writes down a correct or incorrect answer. Process is sound but prone to mistakes.</p>	<ul style="list-style-type: none"> ● Tell me about your thinking. ● Can you explain how you figured out the elapsed time? 	<ul style="list-style-type: none"> ● Could your partner write down your process as you say it? ● Could you use a drawing or representation to prove your thinking? 	See Student E	

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Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions – Teacher Stays to Hear Response <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions – Teacher Poses Question and Walks Away <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> ● <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> ● <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
	<ul style="list-style-type: none"> ● Could you begin by showing me where 1 hour has elapsed in that problem on paper? 	<ul style="list-style-type: none"> ● Is there another method or model that you could use to show the elapsed time? 		
Anticipated Student Response: Student will use a number line or t-chart to go hour by hour and minute by minute or larger increments to produce a correct or incorrect response.	<ul style="list-style-type: none"> ● Tell me about your thinking. ● Does that answer sound like a reasonable amount of time to spend on a bus trip? 	<ul style="list-style-type: none"> ● When you explain to a partner does it still make sense to you? To them? ● Is there another method you could use to prove your thinking? 	See Student A and Student F	
Anticipated Student Response: Student uses more than one efficient method (number line, t-chart, counting up and/or down, etc.) and can give a thorough explanation. A few students may notice the 12-hour relationship between 7:47 am to 7:47 pm and can be much more efficient with their strategy. Some students may be more exacting than stating that Bus A is shorter than Bus B. They might determine that Bus A is a 26-minute shorter trip than Bus B.	<ul style="list-style-type: none"> ● Tell me about your thinking. ● How are your models alike? How are they different? 	<ul style="list-style-type: none"> ● When you explain to a partner does it still make sense to you? To them? ● Which method or strategy that you used was more efficient? What makes it more efficient? 	See Student B and Student D	

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Name _____

Date _____

Bus A or Bus B?

You will take a bus trip across the country to see a professional football game. You have a choice of two buses. (Both bus trips will arrive the day before the game.)

- Bus A departs at 7:47 am and arrives at 8:14 pm.
- Bus B is 12 hours and 53 minutes in length.

Both trips cost \$200. Which bus would you choose, Bus A or Bus B? Why?

Show your thinking using pictures, words, and symbols.

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Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: <ul style="list-style-type: none"> • Uses relationships among mathematical concepts 	<ul style="list-style-type: none"> • Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution 	<ul style="list-style-type: none"> • Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution 	<ul style="list-style-type: none"> • Demonstrates little or no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: <ul style="list-style-type: none"> • Problem solving strategy is efficient 	<ul style="list-style-type: none"> • Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution 	<ul style="list-style-type: none"> • Chooses a problem solving strategy that does not display an understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution 	<ul style="list-style-type: none"> • A problem solving strategy is not evident or is not complete • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: <ul style="list-style-type: none"> • Reasoning is organized and coherent • Consistent use of precise mathematical language and accurate use of symbolic notation 	<ul style="list-style-type: none"> • Communicates thinking process • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to express ideas with precision 	<ul style="list-style-type: none"> • Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking with some imprecision 	<ul style="list-style-type: none"> • Provides little to no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses little or no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: <ul style="list-style-type: none"> • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding 	<ul style="list-style-type: none"> • Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections

Task Supporting Documents

Bus Schedule		
From	To	Time
Richmond, Virginia	New York, New York	8:03 am- 1:45 pm
Richmond, Virginia	Washington, DC	7:45 am- 11:05am
Richmond, Virginia	Virginia Beach, Virginia	8:35am - 10:49am
Washington, DC	Richmond, Virginia	7:30 am- 11:27am
New York, New York	Richmond, Virginia	9:30 pm- 1:02 am
Virginia Beach, Virginia	Richmond, Virginia	3:00 pm- 5:17 pm

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