

Rich Mathematical Task – Grade 5 – *Designing Windows*

Task Overview/Description/Purpose:	
<ul style="list-style-type: none"> • Marisol is measuring angles within a stained glass window. However, her protractor is broken. Using a strategy, find the angle measures of the shaded shape. Can you find more angle measures than Marisol? • In this task, students will investigate interior angles of polygons. By combining and subdividing shapes, students will determine unknown angles in a triangle. • The purpose of this task is for students to use their knowledge of angle measures and polygons to determine unknown angle measures. Students will not be able to use a protractor, so recognizing angle measures as additive is imperative. • Prior to completing this task, students should have experience exploring and measuring angles. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students need to know that a right angle measures exactly 90 degrees, a straight angle measures exactly 180 degrees, and the sum of the interior angle measures of a triangle is 180 degrees. 	
Standards Alignment: Strand – <i>Measurement and Geometry</i>	
<p>Primary SOL: 5.13b Investigate the sum of the interior angles in a triangle and determine an unknown angle measure.</p> <p>Related SOL (within or across grade levels/courses): 4.11, 4.12, 5.12, 5.14b</p>	
<p>Learning Intentions:</p> <ul style="list-style-type: none"> • Content: I am learning to understand the connections between angle measurements and interior angles of a triangle in order to find a missing angle measure. • Language: I am learning how to justify my strategy for finding an unknown angle measurement using appropriate vocabulary (right, straight, degrees). • Social: I am learning how to listen and explain my thinking to my peers in order to move us all forward as mathematical learners. 	
<p>Evidence of Student Learning (based on Essential Knowledge and Skills):</p> <ul style="list-style-type: none"> • I can combine and subdivide polygons to determine angle measurements. • I can determine an unknown angle measure in a triangle by using the relationship of the sum of its interior angles. • I can communicate my mathematical thinking to my peers using language related to measuring and adding interior angles. • I can use at least one representation to support my mathematical thinking. • I can make connections between different representations of mathematical thinking. 	
Mathematics Process Goals	
Problem Solving	<ul style="list-style-type: none"> • Chooses an appropriate strategy to measure angles without using a protractor. • Students will accurately apply their strategy to obtain at least one valid solution.
Communication and Reasoning	<ul style="list-style-type: none"> • Students will communicate their thinking process of finding interior angle measures of a triangle to their peers and their learning community. • Students will justify their solution steps in an organized and coherent manner. • Students will use appropriate mathematical language, including angle names and degrees, to express ideas with accuracy and precision.
Connections and Representations	<ul style="list-style-type: none"> • Students will use at least one appropriate representation to explore and solve the problem and justify their solution. • Students will describe connections between their representations and the representations of their peers.

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Standards Alignment: Strand – *Measurement and Geometry*

- Students will connect and/or extend thinking to other mathematical ideas such as opposite angle congruency and quadrilateral interior angle sums.

Task Pre-Planning

Approximate Length/Time Frame: 60 minutes

Grouping of Students: Teacher should review data collected from SOL 5.12 (classifying angles) and use this information to group students based on current understandings. Groups could be planned to counter misconceptions, move someone along in the sophistication of ideas, or to explore different ways to solve the same problem.

Materials and Technology:

- anchor chart paper
- markers
- copy of task
- pencil, scissors
- grid paper, ruler
- Tangrams

Vocabulary:

- angle measures, degrees
- unknown angle measure
- interior angle
- right angle, straight angle
- sum of the interior angles of a triangle
- acute, obtuse
- isosceles triangle

Anticipate Responses: See Planning for Mathematical Discourse Chart (Columns 1-3)

Task Implementation (Before) 10-15 minutes

Task Launch:

- **Activate prior knowledge:** The teacher will ask students what they already know about angles. Next, the teacher will record student responses on an anchor chart. Ask students how to “draw” what they are describing.

Example:

<p><u>Angles</u></p> <ul style="list-style-type: none">• Right angles measure 90 degrees• Straight angles measure 180

- **Check for student understanding of the task:** The teacher will display the task and read it aloud. Some important ideas to listen for and build upon are:
 - The window is a square.
 - Polygon DEFG is a rectangle.
 - [Share photos of stained glass](#) in order to help students visualize context.
- **Establish clear expectations:** The teacher will display and review the rubric alongside the “I Can” statements. The teacher will ask the questions to make sure the task is understood: “What are we trying to figure out?” “What do you already know that can help you get started?”

Task Implementation (During) 20-30 minutes

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page)
- Select – Teacher will decide which strategies or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation)
- Connect – Teacher will consider ways to facilitate connections between different student responses

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- Students work in purposefully planned groups for 20-25 minutes to explore strategies, share ideas and transfer their ideas to anchor chart paper to share.
 - As teacher is monitoring, teacher will look for strategies that are being used and record on matrix.
 - Teacher should use questions to assess or advance student thinking.
 - Students should be encouraged to explore different strategies for solving and evaluate effectiveness.

Suggestions For Additional Student Support

- Students who need more support using academic language could benefit from sentence stems such as:
 - I agree/disagree with _____'s strategy because _____.
 - The strategy I will use to solve for the interior angle measures is _____.
 - I know the measure of this angle is _____ because _____.
 - To determine the interior angle, I added/subtracted ____.
- Students with weaknesses in memory and vocabulary could benefit from:
 - Anchor chart to reference angle measurements such as right and straight
 - Encouraging students to talk aloud through sequential steps
- Students showing difficulty with visual-spatial skills may benefit from:
 - Using a 4-square problem solving mat (attached)
 - Cutting out enlarged square window to manipulate and fold
- Use motions or other keys to help remember vocab/concepts: 90 degrees- L with fingers, 180= flat hand, interior- inside, acute- small C with fingers compared to the 90 degrees.

Task Implementation (After) 15-20 minutes

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion. Some possible big mathematical ideas to highlight could include:
 - A common misconception
 - Trajectory of sophistication in student ideas (i.e. concrete to abstract; learning trajectories for addition of fractions)
 - Several different solutions for the same task
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
 - How are these strategies alike? How are they different?
 - _____'s strategy is like _____'s strategy because _____
 - How do these connect to our learning goal?
 - Why is this important?
- Highlight student strategies to show the connections, either between different ideas for solutions or to show the connection between levels of sophistication of student ideas.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
 - Students can gallery walk to see all strategies prior to coming together to discuss selected strategies.

Teacher Reflection About Student Learning

- Teacher will use the chart with anticipated student solutions to monitor which students are using which strategies. This will include: possible misconceptions, learning trajectories and sophistication of student ideas, and multiple solution pathways. Next steps based on this information could include:
 - Informing sequence of tasks. What will come next in instruction to further student thinking in fraction computation?
 - Informing small groups based on misconceptions that are not addressed in sharing.
- After task implementation, the teacher will use the Process Goals rubric to assess where students are in process goals. The teacher may decide to focus on one category. Next steps based on this information could include:

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- Informing small groups based on where students are in engagement in the process goal(s) (i.e. think aloud, using specific sentence frames for communication, etc.).

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Planning for Mathematical Discourse

Mathematical Task: Designing Windows

Content Standard(s): 5.13b

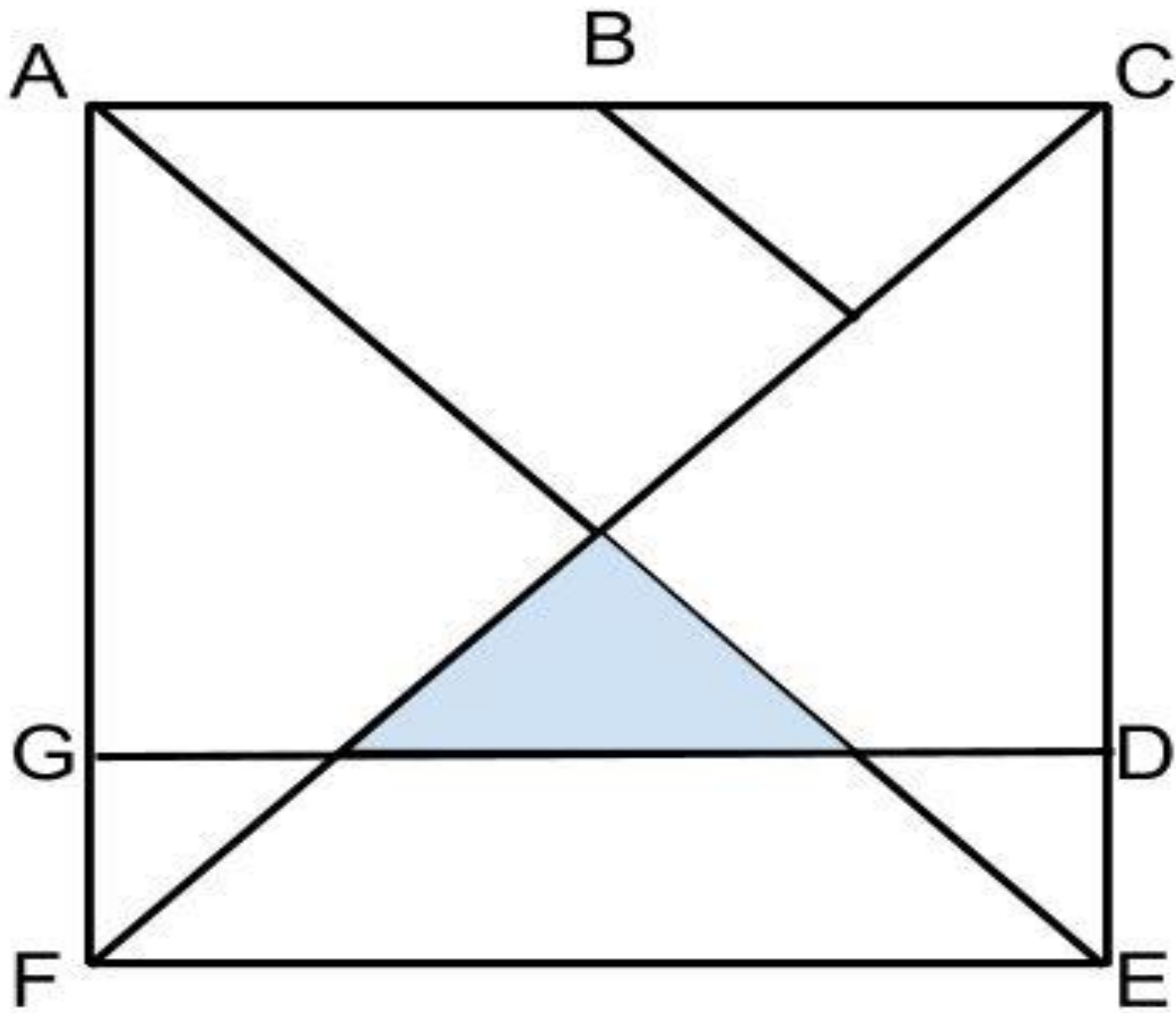
Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation		
Anticipated Student Response/Strategy	Assessing Questions – Teacher Stays to Hear Response	Advancing Questions – Teacher Poses Question and Walks Away	List of Students Providing Response	Discussion Order - sequencing student responses	
<i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	<i>Teacher questioning that allows student to explain and clarify thinking</i>	<i>Teacher questioning that moves thinking forward</i>	<i>Who? Which students used this strategy?</i>	<ul style="list-style-type: none"> ● <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> ● <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i> 	
Anticipated Student Response A: Student recognizes sum of interior angles of a triangle equals 180 degrees. Student divides 360 by 3 to incorrectly determine angle measures of shaded shape as equal (60 degrees).	<ul style="list-style-type: none"> ● Tell me about your thinking. ● Are all three angles in the shaded shape equal? How can you prove they are equal? Not equal? ● Can you label the angle measurements you have proven? 	<ul style="list-style-type: none"> ● How can you use the angle measurements from the other polygons to help you determine the measurements of the shaded shape? 	Student B		
Anticipated Student Response B: Student is able to identify one angle in shaded shape as 90 degrees.	<ul style="list-style-type: none"> ● How could you label that angle? ● What other angles do you notice are 90 degrees? How do you know? Could you label them as well? ● Look at our anchor chart. What do you know about the sum of the angles in a triangle? 	<ul style="list-style-type: none"> ● How can you use your knowledge of the sum of interior angles in a triangle to help you find the measurements of the remaining 2 angles? 	Student A		
Anticipated Student Response C: Student is able to correctly identify interior angles in triangle as 90, 45, and 45 degrees, but unable to demonstrate reasoning with mathematical language.	<ul style="list-style-type: none"> ● Tell me about your thinking. ● Can you label the angles on the diagram? ● Tell me what you did using words like first, next, third 	<ul style="list-style-type: none"> ● I want you to see how group __ is recording their mathematical thinking. How could you improve your justification? 	Student D		

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Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions – Teacher Stays to Hear Response <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions – Teacher Poses Question and Walks Away <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> ● <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> ● <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
	and I will record your thinking.	<ul style="list-style-type: none"> ● What other angle measurements can you determine? Can you record your reasoning? 		
Anticipated Student Response D: Student may come up with 3 measures for the interior angles of the triangle that equal 180 degrees but incorrectly identifies each angle measurement.	<ul style="list-style-type: none"> ● Tell me about your thinking. ● I see that you already know that the sum of the interior angles of a triangle equals 180 degrees. ● Look at the anchor chart we created before the task. What angles from the chart do you see in the window? ● Can you label them? 	<ul style="list-style-type: none"> ● How can you use this information to help you find the unknown angle measures? 	Student E	

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Supplemental Material – Enlarged Window



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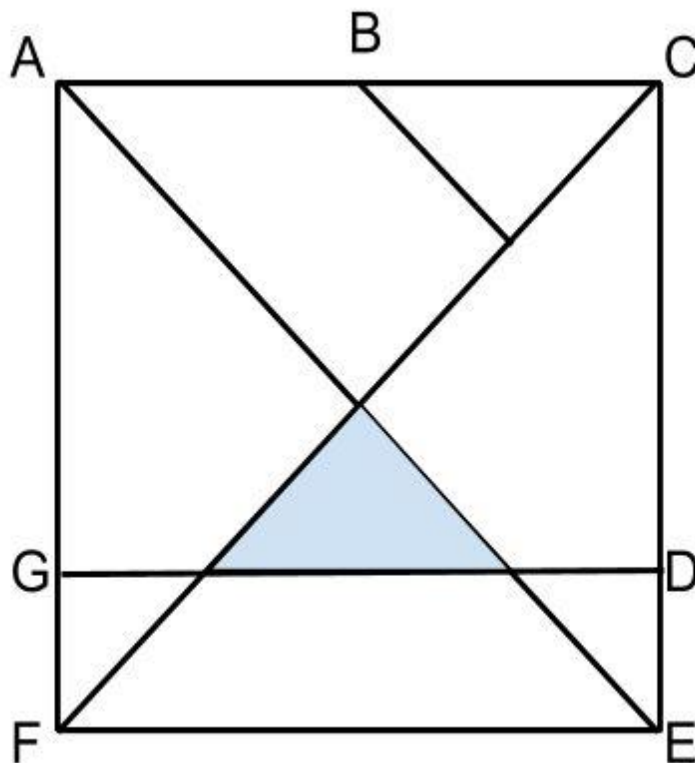
Name _____

Date _____

Designing Windows

Marisol designs windows. She has been asked to create different colored glass for each polygon within the stained glass window below. Marisol needs to measure each angle, but her protractor is broken. She recorded the following facts to help her find the angle measurements:

- Polygon ACEF is a square
- Polygon DEFG is a rectangle



1. Without using a protractor, determine and label the angle measurements for the shaded shape. Describe the strategy you used to determine the angle measurements of the shaded shape.
2. Marisol was able to find 16 angle measurements for the window. Using a strategy, label as many angles as possible. Be prepared to prove your thinking.

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Mathematical Rich Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: <ul style="list-style-type: none"> • Uses relationships among mathematical concepts or makes mathematical generalizations 	<ul style="list-style-type: none"> • Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution 	<ul style="list-style-type: none"> • Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution 	<ul style="list-style-type: none"> • Demonstrates no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: <ul style="list-style-type: none"> • Problem solving strategy is well developed or efficient 	<ul style="list-style-type: none"> • Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution 	<ul style="list-style-type: none"> • Problem solving strategy displays a limited understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution 	<ul style="list-style-type: none"> • A problem solving strategy is not evident • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: <ul style="list-style-type: none"> • Reasoning or justification is comprehensive • Consistently uses precise mathematical language to communicate thinking 	<ul style="list-style-type: none"> • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to communicate thinking 	<ul style="list-style-type: none"> • Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking 	<ul style="list-style-type: none"> • Provides no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: <ul style="list-style-type: none"> • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding 	<ul style="list-style-type: none"> • Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections

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Possible Graphic Organizers

UPSE Problem Solving Mat

<p>UNDERSTAND (What do I know?)</p>	<p>PLAN (What strategy will I use?)</p>
<p>SOLVE</p>	<p>EXPLAIN</p>