Rich Mathematical Task – Grade 6 – *Planning a Dog Park*

Task Overview/Description/Purpose:

- In this task, students will design a fenced in dog enclosure, given a specific amount of fence, to explore measurements (perimeter, circumference, and area) of squares, rectangles, and circles. As an exploration, students will use perimeter, circumference, and area to determine which shape maximizes area when given a set perimeter.
- The purpose of this task is to deepen understanding about the relationship between the area and perimeter of rectangles and the circumference and area of circles.

Standards Alignment: Strand - *Measurement*

Primary SOL: 6.7a The student will

a) solve problems, including practical problems, involving circumference and area of a circle.
b) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.

Related SOL (within or across grade levels/courses): 5.8a, 5.8b, 6.7a, 8.10

Learning Intention(s):

- **Content –** I am learning about the relationship between area and perimeter of rectangles and the area and circumference of circles.
- **Language-** I am learning how to communicate about the relationships and patterns of area, perimeter, and circumference.
- **Social –** I am learning how to explain my strategy and work to others so I can refine my strategies for problem solving.

Success Criteria (Evidence of Student Learning);

- I can determine the area and perimeter of rectangles and the area and circumference of circles in a practical problem.
- I can justify the relationships between area and perimeter using appropriate math language.
- I can make suggestions and utilize suggestions made by my peers to make revisions to my work and thinking.

Mathematics Process Goals

Problem Solving	• Students will apply the concept of perimeter, area, and circumference of rectangles and circles to model a practical situation.
Communication and Reasoning	• Students will justify how they know that their figure has the largest area.
Connections and Representations	• Students will use formulas, tiles, graph paper, or an online application to model their fenced in areas. This task builds upon prior knowledge of perimeter and area.

Task Pre-Planning

Approximate Length/Time Frame: 60 minutes

Grouping of Students: This task would be best completed with students working independently first to find one solution and then putting students in groups of 3-4 to determine which figure and measurements provide the largest area.

Materials and Technology:	Vocabulary:
• graph paper	• perimeter, area

• linking cubes • square tiles • online application like Geogebra or Desmos.com/Geometry • string	• circumference • diameter, radius • length, width • rectangle • circle
Anticipate Responses: See Planning for Mathematical Discourse Chart (Columns 1-3)	

Task Implementation (Before)

Task Launch

- Students will utilize their understanding of area and perimeter from fifth grade to build their understanding. To review these terms, ask students to draw any shape on a piece of paper or white board. Ask them to identify the area and perimeter. Then pair students up with a shoulder-partner and have them talk about the meaning of area and perimeter.
- To help students make sense of the task, it would be beneficial to have students read the task first and circle any words that are confusing to them. You may choose to read the task to students and have them follow along. Then have a class discussion about any words that students circled. Following this discussion, have the students read the task a second time. On the third read through, have students underline any important information.
- Use "snowball" strategy for brainstorming ideas. In groups of 3-5, have students brainstorm possible ideas related to the task. What strategies might they use? How might they show their thinking? Give each group 3-5 minutes to share (roughly 1 minute per student). Then have students form a new group of 3-5. They should not be with any of the same people. Repeat this process of sharing again. "Snowball" is a good way to promote discourse as well as help all students generate and refine strategies for approaching the task. Everyone will come away with at least one idea to get started.

Task Implementation (During)

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page)
- Select – Teacher will decide which strategies or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation)
- Connect – Teacher will consider ways to facilitate connections between different student responses

Suggestions For Additional Student Support *(possible supports or accommodations for individual student, as needed)*
May include, among others:

- Question students, in both assessing and advancing formats, to help students refine their strategies.
- Have all of the manipulatives out on a central table so that students can get what they need, as they need it.
- Some students get stuck at just making one model that fits the requirements. Asking questions like "How confident are you?" and "What would convince someone?" will help students get past this point.
- For students with motor processing problems, allow him/her to communicate the reasoning in other ways such as video recording or typing answers.

- For students with attention challenges ask student to restate the problem or important information. You may choose to limit the types/quantity of manipulatives the student has access to.
- For students with fluid reasoning weaknesses, provide the graphic organizer in this document to help organize their thinking.

- For students who need academic language support, consider the use of a visual word wall or reference sheet for students to use identifying length, width, rectangle, area, perimeter, and square. (e.g., yards is a multiple meaning word, backyard, dimensions)
    - Provide images of the different possible shapes, fence, dog house, etc.)
- For students who need more support in justifying their thinking, you may choose to provide them with the sentence frames below.
    - What I know about the problem is…
    - My method for solving the problem was…
    - I know that my model has the largest area for the dog to play because…
- For ELs with first language literacy, try to provide prompt, or parts of prompt, in their home language.
- In instruction, use motions to emphasize “maximum”, more/most/largest space.

Task Implementation (After) 20 minutes

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion. Some possible big mathematical ideas to highlight could include:
    - Common misconceptions
    - Concrete to representational to abstract and/or rectangles to squares to circles
- Connect different students’ responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
    - How are these strategies alike? How are they different?
    - Where do you see _____’s strategy in ______’s strategy?
    - What conclusions can you draw about the relationship between perimeter/circumference and area?
    - Why is this important?
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion. Some possible ways to do this are to-
    - Assign roles like time keeper, task master, material fetcher, and recorder of strategies to each member of the group.
    - “Snowball” in the middle of the task to have students get new ideas/refine their thinking. For “snowball” strategy directions see “Task Implementation” above.

Teacher Reflection About Student Learning:

- Teacher should use the chart on the next page with the anticipated student solutions to monitor which students are using each strategy as well as record any additional strategies encountered. The sequence of tasks will inform what will come next in instruction to further student ideas and thinking. Form small groups to address misconceptions that are not addressed in class debrief.
- Use the Process Goals Rubric to assess the level of student ability related to the process goals. Information gathered from this rubric could identify small groups for later instruction, identifying specific students to partner with one another, and/or identifying students who need more teacher modeling and think alouds.

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Planning for Mathematical Discourse

Mathematical Task: ______________________________ Content Standard(s): ____________________________________

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions:** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions:** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response: I don't know what to do?	• What is the question asking you? • What information do you have? • What do you notice? • What do you wonder? • What do you predict the solution might look like?	• What is a strategy you heard when you were in a group? • What could you use to model the problem? • What do you predict the solution might look like?		
Anticipated Student Response: MISCONCEPTION-Use 72 as area	• What would a fence look like? • Is 72 yards the area or perimeter? How do you know? • Let's go back to the question for a second. Is everything still making sense?	• How can you use your knowledge of perimeter to rearrange/rethink your model?		

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Anticipated Student Response: MISCONCEPTION-Use 72 linking cubes to make the perimeter and then using the exterior of the blocks as the perimeter. This results in counting the corner cube twice.	• Tell me about your thinking. • How long is each side of your figure? What does perimeter mean?	• Does your length and width check out?	**Student F**	
Anticipated Student Response: Use dimensions of the yard as the length and/or width	• Let's go back to the question for a second. Is everything still making sense? • How long is each side of your figure? What does perimeter mean?	• Does your length and width check out?	**Student D**	
Anticipated Student Response: Any rectangle with a perimeter of 72 (ie. 20x16, 17x19, etc.)	• How confident in your answer are you? • What would convince you that you found the largest area?	• How can you confirm your area is the greatest?		

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Anticipated Student Response: Divide 72(perimeter) by 4 and then square 18 to find the area. Answer= (324 yd^2)	• Tell me about your thinking. • What method did you use? • How do you know this is the largest area?	• What conjecture can you make about the relationship between perimeter and area?	**Student A** **Student B** **Student C** **Student E**	
Anticipated Student Response for Sequel: Use string and graph paper to estimate the area of a circle.	• Tell me about your thinking. • What method did you use? • How do you know this is the largest area?	• Can you make another representation that meets the requirements?	**Student B** **Student E**	
Anticipated Student Response for Sequel: Divide 72 (circumference) by 2 times 3.14 to find the radius. Then square the radius and multiply by 3.14 for answer of (412.74 yd^2)	• Tell me about your thinking. • What method did you use? • How do you know this is the largest area?	• What conjecture can you make about the relationship between perimeter/ circumference and area?		

Planning a Dog Park

Name______________________________________ Date_________________________________

Your parents have asked you to design an enclosed area in your backyard for your dog.

- The enclosed area can be in the shape of a square or a rectangle.
- The area will be enclosed with a fence that cannot be attached to another structure (i.e., the house, shed, etc.).
- There is 72 yards of fencing available.
- The dimensions of your rectangular backyard is 30 yards by 35 yards.

What is the largest area in your backyard that can be enclosed for your dog? What are the dimensions of this enclosed area? Justify how you know that your design provides the largest area.

Sequel: What is the largest area in your backyard for your dog if the enclosure can be a circle? How does this change your answer?

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Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: • Uses relationships among mathematical concepts	• Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution	• Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	• Demonstrates little or no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: • Problem solving strategy is efficient	• Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution	• Chooses a problem solving strategy that does not display an understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	• A problem solving strategy is not evident or is not complete • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: • Reasoning is organized and coherent • Consistent use of precise mathematical language and accurate use of symbolic notation	• Communicates thinking process • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to express ideas with precision	• Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking with some imprecision	• Provides little to no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses little or no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: • Uses representations to analyze relationships and extend thinking Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	• Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections

Dog Park Possible Solutions

Length	Width	Perimeter	Area