Rich Mathematical Task – Grade 7 – *Summer Passes*

Task Overview/Description/Purpose:

- In this task students will explore additive and multiplicative relationships in order to deepen their understanding of the differences between these two mathematical representations. Students will analyze three different earning relationships to determine which one will earn money the fastest, how long will it take for all three relationships to reach the amount needed for a summer pass, and when Michael, Susie and Karl can all get their summer pass.
- The purpose of the task is to deepen understanding by connecting rates, tables, and equations while, at the same time, contrasting proportional and additive relationships.

Standards Alignment: Strand - Patterns, Functions and Algebra

Primary SOL: 7.10e The student will

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Related SOL (within or across grade levels/courses): 6.12, 8.16

Learning Intention(s):

- **Content (based on Essential Knowledge and Skills)** - I am learning to recognize proportional and additive relationships expressed verbally and represent them as tables, graphs or equations. I am learning to make connections between different representations.
- **Language**- I am learning to use mathematics vocabulary (rate of change, proportional relationship, and additive relationship) appropriately when solving problems.
- **Social** - I am learning to explain my problem-solving thinking clearly to my peers.

Success Criteria (Evidence of Student Learning):

- I can recognize proportional and additive relationships when expressed verbally as well as in table, graph or equation form.
- I can make connections between the different representations of a relationship (verbal, table, graph, and equations) using language such as (the equation…, but the graph shows…. differently…).
- I can use my mathematics vocabulary appropriately when explaining my thinking process to my peers. (e.g., ratio 1 is a faster rate of change because…).

Mathematics Process Goals

Problem Solving	• Students will analyze relationships in order to compare outcomes. They will use different representations (tables, graphs, etc.) to compare situations and determine the fastest growing relationship.

Communication and Reasoning	• Students will justify their solutions verbally and with mathematical evidence. • Students will communicate the outcomes in a practical situation and support their thinking with evidence.
Connections and Representations	• Students will make sense of the impact of rates in different relationships. • Students will illustrate mathematical relationships using verbal descriptions, tables, graphs and equations.

Task Pre-Planning

Approximate Length/Time Frame: *45-60 minutes*

Grouping of Students:

- Large group. Set the stage through a series of questions. Be sure that vocabulary has been presented in previous lessons. Set expectations for each part of the task. Present task.
- Independent Think Time: give students 2 minutes to think about the task. NO PAPER AND PENCIL.
- Shoulder Partners: give students 2 minutes to share their ideas of how to approach. NO PAPER AND PENCIL.
- Independent - Complete task.
- Large Group. In the closing, Making Connections Discussion, consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion. You will select individuals to share his/her choice of table, rule, equation, and graph during the implementation stage.

Materials and Technology:

- Graphic organizers
- Graph paper
- Desmos graphing calculator

Vocabulary

- Initial amount
- constant
- rate/rate of change
- slope
- y-intercept
- proportional relationship
- additive relationship

Anticipate Responses: See Planning for Mathematical Discourse Chart (columns 1-3)

Task Implementation (Before)

Task Launch:

- In large group facilitate student talk using Pair Share with the following questions:
  - What are proportional (multiplicative) relationships? What are additive relationships? How are they different from each other?

- How could you represent a proportional relationship? How about an additive one?
- What might these relationships have in common?
- How do you demonstrate your mathematical understanding? How will the teacher understand your thinking? What does it mean to justify your reasoning? What is important about mathematical models?
- Read the task together using the Three Read Protocol. (Read three times with a different focus each time. First focus is comprehending the text (context). Second focus is comprehending the mathematical structure of the situation. The third focus is listing all the possible mathematical questions)

- Provide 2 minutes of Think Time for students. Explicitly state, "Students DO NOT write anything down at this time."
- Provide 2 minutes for Shoulder Partners to share their ideas. Let students know that they should NOT write anything down at this time as well.
- Ask if there are any other questions, provide 10-15 minutes for students to complete the task independently.

Task Implementation (During)

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page)
- Select – Teacher will decide which strategies or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning. Teacher will look for students who have examples of well-reasoned logic, tables, graphs, and/or equations.
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation) during the closure discussion.
- Connect – Teacher will consider ways to facilitate connections between different student responses.

Suggestions for Additional Student Support *(possible supports or accommodations for individual student, as needed)*

- Ask leading questions for the student who cannot get started.
- Ask and answer questions about unknown words.
- For students who have difficulty with visual motor skills, direct them to the graphic organizers. Graphic organizers should be kept with the hands-on manipulatives. **Graphic organizers should not be handed out with the task.** Graphic organizers should be available at student request. Teacher may choose to distribute graphic organizers to students as needed.
- Students who need more language support could benefit from visual word walls or small glossary (e.g., pass for amusement park as they may not have this background knowledge, earn money=work)
- For ELs with first language literacy, try to provide prompt, or parts of prompt, in their home language.
- Read the prompt aloud.
  - Use motions with visuals to clarify the options (pulling weeds, cleaning, earning money).
- For students who need support in justifying their thinking, you may choose to provide them with the sentence frames below.
  - What I know about the problem is…

- My method for solving the problem was…
- To set up the three ratios, I first…

Task Implementation (After)

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion. Suggested sequence would be tables, rules, equations, graphs and mixed representations.
- Lead the class discourse to identify the similarities and differences between the multiplicative/proportional relationship and the additive relationships.
- Connect different students' responses to the multiplicative and additive relationships. In large group discussion, draw out the connections between the representations to the key mathematical ideas such as how a multiplicative relationship will grow faster than an additive relationship.
- Connect the rate of change to the tables. Connect the slope and y-intercept of the graphs to the relationship. Discuss how the rate in the table and the slope of the line show that the multiplicative relationship grows faster.
- After the group discussion, have students complete an exit ticket to explain the differences between the proportional and additive relationships or relate the rate in the table to the slope of the graphs.

Teacher Reflection About Student Learning:

- Were the instructional objectives met? Were the students able to differentiate between additive and multiplicative relationships by the end of the lesson?
- Were the process goal objectives met? Were students able to explain their work verbally (oral or written)? Does vocabulary need further development?
- Were the students productively engaged?
- Was enough support provided during the task using the chart of anticipated responses? Did additional responses occur that were not anticipated?
- What strategies did the students struggle with the most? Were there reoccurring student misconceptions?
- How will the evidence provided through student work inform further instruction?
- Did the task rubric assist in identifying students who need additional support? What additional assistance and support will be needed for students who are developing or emerging?

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Planning for Mathematical Discourse

Mathematical Task: _Summer Passes_____ Content Standard(s): _7.10e_____

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response: No approach. Student has no idea where to begin. *"I don't know what to do."*	• How would you describe the problem in your own words? • What do you know that is not stated in the problem? • What facts do you have?	• Can you use a representation to explain what you are thinking?	**Student F**	
Anticipated Student Response: Logical Thinking Approach "I just know that the answer is…"	• How do you know? • Walk me through your steps. Where did you begin?	• Use a mathematical model to represent your thinking? • What math terms can you use in your	**Student B**	

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
	• Do you notice any patterns?	justification of your solution?		
Anticipated Student Response: Table Only Approach	• What are your input values representing? What is the output value representing? • Is your table labeled appropriately? • Can you model the situations with some type of manipulative for each day?	• Can you think of another way to justify your solutions? • What math terms can you use in your justification of your solution? • How is the rate represented in the table?	**Student C**	
Anticipated Student Response: Graph Only Approach	• Is there another way to represent the graph? • How would you determine the day	• Can you think of another way to justify your solutions? • What math terms can you use in your	**Student E**	

Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas.* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
	that Michael, Susie, and Karl can quit working? • What line would represent the cost of the pass?	justification of your solution? • What part of the graph represents the slope?		
Anticipated Student Response: Equation Only Approach	• Now that you have the equations, how will you determine when they will have enough money to buy the passes?	• Can you think of another way to justify your solutions? • What math terms can you use in your justification of your solution? • How are the equations for alike and different for the three situations?	**Student A** **Student D**	

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Student Version of Task Description

Michael, Susie, and Karl plan to purchase summer passes to a local amusement park. They decide to work summer jobs to earn the money for the summer passes.

- Michael will babysit his little brother every day. He will earn $10 a day for babysitting.

- Susie's sister has a summer lawn care business. Susie will help her sister pull weeds out of the flower beds every day. Her sister will pay her $50 upfront and then a dollar each day.
- Karl has already saved $35 for his summer pass. His mom agrees to give him one dollar for every day that he does his chores. Karl does his chores every day.

If a summer pass to the amusement park costs $86, who will be the first one to have enough money to buy the pass? How long will it be before they can all go together? Explain your reasoning and give evidence of your position.

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	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	***Proficient Plus:*** ● Uses relationships among mathematical concepts or makes mathematical generalizations.	● Demonstrates an understanding of concepts and skills associated with task ● Applies mathematical concepts and skills which lead to a valid and correct solution	● Demonstrates a partial understanding of concepts and skills associated with task ● Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	● Demonstrates no understanding of concepts and skills associated with task ● Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	***Proficient Plus:*** ● Problem solving strategy is well developed or efficient	● Problem solving strategy displays an understanding of the underlying mathematical concept ● Produces a solution relevant to the problem and confirms the reasonableness of the solution	● Problem solving strategy displays a limited understanding of the underlying mathematical concept ● Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	● A problem solving strategy is not evident ● Does not produce a solution that is relevant to the problem
Communication and Reasoning	***Proficient Plus:*** ● Reasoning or justification is comprehensive ● Consistently uses precise mathematical language to communicate thinking	● Demonstrates reasoning and/or justifies solution steps ● Supports arguments and claims with evidence ● Uses mathematical language to communicate thinking	● Reasoning or justification of solution steps is limited or contains misconceptions ● Provides limited or inconsistent evidence to support arguments and claims ● Uses limited mathematical language to partially communicate thinking with some imprecision	● Provides no correct reasoning or justification ● Does not provide evidence to support arguments and claims ● Uses no mathematical language to communicate thinking
Representations and Connections	***Proficient Plus:*** ● Uses representations to analyze relationships and extend thinking ● Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	● Uses a representation or multiple representations, with accurate labels, to explore and model the problem ● Makes a mathematical connection that is relevant to the context of the problem	● Uses an incomplete or limited representation to model the problem ● Makes a partial mathematical connection or the connection is not relevant to the context of the problem	● Uses no representation or uses a representation that does not model the problem ● Makes no mathematical connections

Possible Graphic Organizers

