

Rich Mathematical Task – Grade 8 –How Many Stones Will I Need?

Task Overview/Description/Purpose:	
<ul style="list-style-type: none"> Students view a table that displays a growing pattern in which the number of stones needed to surround a garden increases as the size of the garden increases. In this task, the expectation is that students develop algebraic expressions to represent the width, length, and number of stones needed as well as communicate the connection between the expressions and the garden itself. In this task, students will create, simplify, and evaluate algebraic expressions. This task allows students to produce expressions that reflect different ways of representing the pattern posed while, simultaneously, allowing students to see that each expression will simplify to the same equivalent expression. 	
Standards Alignment: Strand – Patterns, Functions, and Algebra	
Primary SOL: 8.14 The student will <ul style="list-style-type: none"> a) evaluate an algebraic expression for given replacement values of the variables; and b) simplify algebraic expressions in one variable 	
Related SOL (within or across grade levels/courses): 5.19a, 5.19c, 7.11, 8.16, 8.17, 8.18	
Learning Intention(s):	
<ul style="list-style-type: none"> Content – I am learning about representing patterns as an algebraic expression as well as simplifying and evaluating my algebraic expressions. Language – I am learning how to justify and explain my thinking while creating algebraic expressions to represent a pattern. Social – I am learning to collaborate with my classmates to hear each other’s views of a pattern and to validate one another’s work. 	
Success Criteria (Evidence of Student Learning);	
<ul style="list-style-type: none"> I can create algebraic expressions that model a real-life situation. I can simplify algebraic expressions in one variable. I can use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for the given replacement values of the variables. I can justify the thinking that I applied when creating and simplifying algebraic expressions to match information presented in a practical situation to my classmates. I can benefit from considering other expressions created by my classmates that model the same real-life situation in a different way. 	
Mathematics Process Goals	
Problem Solving	<ul style="list-style-type: none"> Students will apply their knowledge of creating, simplifying, and evaluating algebraic expressions to a practical problem that a consumer encounters as he/she considers the number of stones needed to surround a garden.
Communication and Reasoning	<ul style="list-style-type: none"> Students explain how algebraic expressions that they create relate to a growing pattern presented pictorially. They will explain the reasoning applied to develop the expressions as well as demonstrating, algebraically, how to simplify each expression provided.

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Standards Alignment: Strand – Patterns, Functions, and Algebra	
Connections and Representations	<ul style="list-style-type: none"> Students will connect a visual pattern to a table of values which results in a set of algebraic expressions. They will make connections between varying expressions created. Students will validate the accuracy of each simplified expression by connecting them back to the values in the tables and pictorial representations.
Task Pre-Planning	
Approximate Length/Time Frame: 50 minutes	
<p>Grouping of Students: The task will begin with a whole class grouping as it is distributed to students so that it can be read aloud. After the class, as a whole, has gone through an initial read, students will be given an opportunity to ask clarifying questions. Then, students will read through one more time individually. After they have finished this second read, the teacher should give students one more opportunity to ask clarifying questions before allowing them to begin working on an individual basis. As the students are working on the task, the teacher will be circulating to look for a variety of strategies being applied to the task. The teacher should guide students as needed and alert students who are using different strategies that he/she plans to have them share their work with the class at the conclusion of the task. In order to reflect and move forward, the grouping will return to whole class so that the teacher can orchestrate sharing out of strategies which will allow students to communicate their reasoning and find that each expression for the number of stones needed resulted in the same final product.</p>	
<p>Materials and Technology:</p> <ul style="list-style-type: none"> the task itself any manipulative that students might use to simulate the task calculators (Desmos or handheld) 	<p>Vocabulary:</p> <ul style="list-style-type: none"> variable algebraic expression coefficient constant simplify Distributive Property terms like terms evaluate replacement value
Anticipate Responses: See Planning for Mathematical Discourse Chart (Columns 1-3)	
Task Implementation (Before)	
<p>Task Launch</p> <ul style="list-style-type: none"> As a whole class, the teacher should orchestrate a read aloud of the task. After the initial reading, teachers should allow students to ask any questions that they may have about the task itself or the vocabulary presented. The students should be provided with a second opportunity to read through the task on an individual basis. After this second reading, teachers should provide students with one more opportunity to ask clarifying questions. The teacher must discourage any discussion of strategy at this point. 	
Task Implementation (During)	
<p>Directions for Supporting Implementation of the Task</p> <ul style="list-style-type: none"> Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see chart on next page) 	

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- Select – Teacher will decide which strategies or thinking that will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation) during the closure discussion.
- Connect – Teacher will consider ways to facilitate connections between different student responses

Suggestions For Additional Student Support

- Some students with visual-motor weaknesses may benefit from graph or lined paper to help them organize their work.
- Students with weaknesses in memory and language could benefit from word walls or graphic organizers to activate prior knowledge about algebraic expressions.
- Provide manipulatives such as color tiles to support visual-spatial-kinesthetic learning.
- Post visual cues such as copies of the gardens pictured for students who need support with memory.
- For students who need support in justifying their thinking, you may choose to provide them with the sentence frames below.
 - What I know about the problem is...
 - My method for solving the problem was...
 - To simplify expressions, I first...

Task Implementation (After)

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion.
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion. For instance, provide the students whose work was not selected as part of the sequence of student work that will be shown the opportunity to validate and/or question what they see.

Teacher Reflection About Student Learning:

- Student understanding of the content through the use of the process goals will be assessed with the Rich Mathematical Task Rubric.
- Students have a variety of ways to enter this task. Teachers will need to anticipate these different entry points but also be prepared for others. The key here is allowing the students to communicate their reasoning as they are working through the task.
- The results of this task will help the teacher assess background knowledge and give the students an opportunity to apply this knowledge to a new situation.
- Teachers should use this task to highlight how each student's entry point was valid as they all simplify to the same final expression.

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Planning for Mathematical Discourse

Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions: <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions: <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> • <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> • <i>Connect different students' responses and connect the responses to the key mathematical ideas.</i> • <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
Anticipated Student Response: Non-starter	<ul style="list-style-type: none"> • What are you thinking? • Is there anything that you need me to clarify about the task? 	<ul style="list-style-type: none"> • Can you draw pictures of the gardens on graph paper to see if that might help? • Can you build models of the gardens with color tiles to see if that will help you get started? 		
Anticipated Student Response: Student represents the length and width of the actual garden with appropriate algebraic expressions and simplifies correctly	<ul style="list-style-type: none"> • Can you explain the thought process you went through to create each of the algebraic expressions that you have recorded? • Will you show me how your expressions connect to the pictures of the gardens? 	<ul style="list-style-type: none"> • Can you connect your simplified expression back to the picture of the garden? • I saw another student who started with different algebraic expressions than you. You both have the same simplified expression. Can you figure out what he/she might have done? 	Student F	

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Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions: <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions: <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> • <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> • <i>Connect different students' responses and connect the responses to the key mathematical ideas.</i> • <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
Anticipated Student Response: Student represents the length and width of the garden and stepping stones together with an appropriate algebraic expression and simplifies correctly	<ul style="list-style-type: none"> • Can you explain the thought process you went through to create each of the algebraic expressions that you have recorded? • Will you show me how your expressions connect to the pictures of the gardens? 	<ul style="list-style-type: none"> • Can you connect your simplified expression back to the picture of the garden? • I saw another student who started with different algebraic expressions than you. You both have the same simplified expression. Can you figure out what he/she might have done? 	Student A Student B	
Anticipated Student Response: Student represents all three expressions appropriately, without connecting the three, but still simplifies correctly	<ul style="list-style-type: none"> • Can you explain the thought process you went through to create each of the algebraic expressions that you have recorded? • Will you show me how your expressions connect to the pictures of the gardens? 	<ul style="list-style-type: none"> • Can you connect your algebraic expression for the total number of stones needed to your expressions for the length and width of the garden? • Can you connect your simplified expression back to the picture of the garden? 	Student E	

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Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions: <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions: <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> • <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> • <i>Connect different students' responses and connect the responses to the key mathematical ideas.</i> • <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
Anticipated Student Response: Student represents the length and width appropriately but has difficulty writing an expression for the total number of stones needed	<ul style="list-style-type: none"> • Can you explain the thought process you went through to create your algebraic expressions for the number of stones wide and long? • What is keeping you from writing an expression for the total number of stones needed? 	<ul style="list-style-type: none"> • Do you see any pattern in the total number of stones needed? • Did you use the number of stones wide and long to help you determine the total number of stones needed? • The first garden uses 10 stones. We could write an expression of $10n$, but that doesn't work for the other entries in the table. Can you move down the table and create other possible expressions that might work with all of the gardens in the table? 		
Anticipated Student Response: Student represents the three expressions appropriately, but	<ul style="list-style-type: none"> • Can you explain the thought process you went through to create each of the algebraic expressions that you have recorded? 	<ul style="list-style-type: none"> • Your reasoning is on-target. I would like to share your thinking with the class; however, the mathematics you have 		

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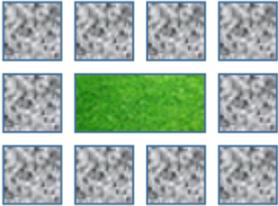
Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions: <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions: <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> • <i>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</i> • <i>Connect different students' responses and connect the responses to the key mathematical ideas.</i> • <i>Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</i>
he/she has applied an area application whose simplification is beyond the scope of the Math 8 curriculum.	<ul style="list-style-type: none"> • Will you show me how your expressions connect to the pictures of the gardens? 	created requires some Algebra I content knowledge. Can you think of another way to represent the total number of stones needed that does not require us to multiply algebraic expressions?		
Anticipated Student Response: Student represents the three expressions appropriately, but he/she does not know how to simplify the expression(s) created.	<ul style="list-style-type: none"> • Can you explain the thought process you went through to create each of the algebraic expressions that you have recorded? • Will you show me how your expressions connect to the pictures of the gardens? • Can you tell me what you know about like terms? • What does it mean when someone asks you to apply the Distributive Property? 	<ul style="list-style-type: none"> • Can you use a manipulative to model the expressions that you have created? • Think about how many sets of “n” there are in your algebraic expression. 	Student C Student D	

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Name _____ Date _____

How Many Stones Will I Need?

The illustration below shows how a gardening company defines the sizes of its gardens as well as how it puts stepping stones around the outside of each.

size	pattern
1	
2	
3	

- 1.) Fill the table below with values for gardens of given sizes. Then, generalize by writing an algebraic expression that could be used to calculate the number of stones needed for a garden of size n .

Garden Size	# of Stones Wide	# of Stones Long	# of Stones Needed
1			
2			
3			
4			
5			
10			
n			

- 2.) Explain how the three algebraic expressions that you created in the bottom row relate to the gardens.

3.) Are your algebraic expressions in their simplest form? If not, simplify them in the space below.

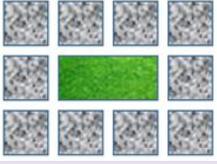
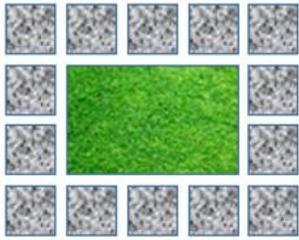
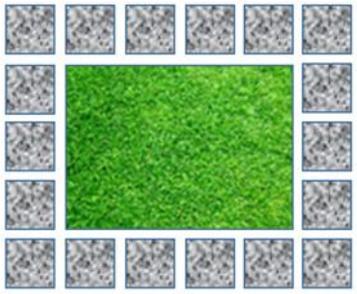
4.) Use your simplified expression for the number of stones needed to evaluate and determine how many stones are required for a garden of size 100. Does the result make sense? Explain your reasoning.

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	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	<p>Proficient Plus:</p> <ul style="list-style-type: none"> Uses relationships among mathematical concepts 	<ul style="list-style-type: none"> Demonstrates an understanding of concepts and skills associated with task Applies mathematical concepts and skills which lead to a valid and correct solution 	<ul style="list-style-type: none"> Demonstrates a partial understanding of concepts and skills associated with task Applies mathematical concepts and skills which lead to an incomplete or incorrect solution 	<ul style="list-style-type: none"> Demonstrates little or no understanding of concepts and skills associated with task Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	<p>Proficient Plus:</p> <ul style="list-style-type: none"> Problem solving strategy is efficient 	<ul style="list-style-type: none"> Problem solving strategy displays an understanding of the underlying mathematical concept Produces a solution relevant to the problem and confirms the reasonableness of the solution 	<ul style="list-style-type: none"> Chooses a problem solving strategy that does not display an understanding of the underlying mathematical concept Produces a solution relevant to the problem but does not confirm the reasonableness of the solution 	<ul style="list-style-type: none"> A problem solving strategy is not evident or is not complete Does not produce a solution that is relevant to the problem
Communication and Reasoning	<p>Proficient Plus:</p> <ul style="list-style-type: none"> Reasoning is organized and coherent Consistent use of precise mathematical language and accurate use of symbolic notation 	<ul style="list-style-type: none"> Communicates thinking process Demonstrates reasoning and/or justifies solution steps Supports arguments and claims with evidence Uses mathematical language to express ideas with precision 	<ul style="list-style-type: none"> Reasoning or justification of solution steps is limited or contains misconceptions Provides limited or inconsistent evidence to support arguments and claims Uses limited mathematical language to partially communicate thinking with some imprecision 	<ul style="list-style-type: none"> Provides little to no correct reasoning or justification Does not provide evidence to support arguments and claims Uses little or no mathematical language to communicate thinking
Representations and Connections	<p>Proficient Plus:</p> <ul style="list-style-type: none"> Uses representations to analyze relationships and extend thinking <p>Uses mathematical connections to extend the solution to other mathematics or to deepen understanding</p>	<ul style="list-style-type: none"> Uses a representation or multiple representations, with accurate labels, to explore and model the problem Makes a mathematical connection that is relevant to the context of the problem 	<ul style="list-style-type: none"> Uses an incomplete or limited representation to model the problem Makes a partial mathematical connection or the connection is not relevant to the context of the problem 	<ul style="list-style-type: none"> Uses no representation or uses a representation that does not model the problem Makes no mathematical connections

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Possible Graphic Organizer

Garden Size	Pattern	# of Stones Wide	# of Stones Long	# of Stones Needed
1				
2				
3				
4				
5				
10				
<i>n</i>				