Rich Mathematical Task – Algebra – *Shopping Translations*

Task Overview/Description/Purpose:

- In this task, students will plan for expenses that can occur when going to college. The expenses will be represented as verbal expressions that students will represent algebraically in order to determine whether they can afford the expenses based on the money they earned while working a summer job. If students cannot afford all the expenses, they will need to decide which items they would purchase and explain why.
- The purpose of this task is for students to represent verbal quantitative situations algebraically and to evaluate those expressions for the given replacement value of the variable, the money they earned at their summer job.

Standards Alignment: Strand – *Expressions and Operations*

Primary SOL: A.1 The student will

a) represent verbal quantitative situations algebraically; and
b) evaluate algebraic expressions for given replacement values of the variables.

Learning Intention(s):

- **Content -** I am learning how to represent verbal quantitative situations algebraically and evaluate algebraic expressions for given replacement values of the variables.
- **Language -** I am learning to explain and justify my thinking and reasoning when determining the value of algebraic expressions for given replacement values of the variables.
- **Social -** I am learning how to communicate my mathematical thinking to my peers and ask probing questions that help my peers and me advance our thinking representing verbal quantitative situations algebraically and evaluating expressions.

Success Criteria (Evidence of Student Learning):

- I can represent practical situations with algebraic expressions in a variety of representations.
- I can evaluate algebraic expressions for given replacement values of the variables.

Mathematics Process Goals

Problem Solving	• Students will use the real-world scenario to represent verbal quantitative situations algebraically. They will then use the value of the money they earned at their summer job to use as a replacement value in their algebraic expression to evaluate the expression. • Students will determine whether they have enough money to afford the items that are needed or wanted for their first year of college.
Communication and Reasoning	• Students will support their decision of which items they purchased based on the sum of the algebraic expressions that represent the college expenses. • Students will use mathematical language to justify their reasoning.
Connections and Representations	• Students will represent verbal quantitative situations algebraically. • Students will make connections that are relevant to the context of the problem using algebraic expressions.

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Task Pre-Planning

Approximate Length/Time Frame*:* 60 minutes

Grouping of Students: Consider using a digital platform to have students to provide evidence of their prior knowledge of representing practical situations with mathematical symbols. Students who can accurately represent sum, product, difference, quantity, and quotient given a variety of verbal expressions could be grouped together and students who are not able to accurately represent the vocabulary mathematically could be grouped together. Also consider grouping students based on language or social needs. When completed with the task, ask students to share their solutions with another individual or team to determine the validity of their task. Finally, create small groups of varying ability levels to share and discuss which items were purchased and why at the conclusion of the task.

Materials and Technology:

- Copy of task
- Pencil/paper or electronic platform (Google document) for typing and submitting work
- Desmos activity (optional)
- Sample Graphic Organizer (for Mathematical Operations and Symbols (attached))
- Graphing utility

Vocabulary:

- algebraic expressions
- evaluate
- symbolic representation
- variables
- sum
- quotient
- square root
- fraction of
- difference
- quantity
- less than

Anticipate Responses: See the Planning for Mathematical Discourse Chart (columns 1-3).

Task Implementation (Before) 15 minutes

Task Launch:

- Consider a number talk or brainstorming session to launch the task and assess students' abilities of ways to represent vocabulary mathematically.
    - The optional Desmos activity can be used to monitor and assess students' prior knowledge of vocabulary for translating algebraic expressions. Students may need assistance with this platform, so consider spending time to support students prior to using Desmos to ensure they understand how to use the platform.
    - The Mathematical Operations and Symbols graphic organizer could be used to measure the depth of vocabulary that students know in order to group students appropriately.
- Consider using the suggestions for grouping of students above to group students appropriately. Monitor groups to support students working collaboratively.
- Share learning intentions and success criteria with students prior to starting the task.
- Have students answer the following questions:
    - What are some items that are *wants* and what are some items that are *needs?*
    - What are some items you think you might *need* when you go to college?
    - What are some items you think you might *want* when you go to college?
- Students will use underlining, highlighting, cue words, or a visual vocabulary word wall to help make sense of the task.
- Students will access prior knowledge and vocabulary regarding representing verbal quantitative situations using the VDOE Word Wall Cards. Have students create a Frayer model for words they have not mastered.

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Task Implementation (During) 45 minutes

Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see the Planning for Mathematical Discourse chart on next page).
- Select – Teacher will decide which strategies will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning.
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation).
- Connect – Teacher will consider ways to facilitate connections between different student responses.
    - Students work in purposefully planned groups for 20-25 minutes to explore strategies, share ideas and transfer their ideas to paper using pictures, words, and symbols.
        - As the teacher is monitoring, teacher will look for strategies that are being used and record on Planning Chart.
        - The teacher should use questions to assess or advance student thinking.
        - Students should be encouraged to explore different strategies for solving and evaluate effectiveness.

Suggestions For Additional Student Support

- Possible use of sentence frames to support student thinking:
    - I chose _____ as items I would purchase because …
- Possible actions to support vocabulary development:
    - Have students complete a Frayer model.
    - Display VDOE Word Wall Cards.
    - Have students create a math vocabulary book for mathematical operations and symbols.
    - Ask students to speak with each other about words that can be used for addition, subtraction, division, and multiplication.
    - Possible use of word associations and pneumonic devices.
    - Consider sharing with students that the word quotient has a Latin background meaning “how many times” and can be used to figure out “how many times” the divisor goes into the dividend.
    - Consider exploring word associations and derivations of mathematical words for students to make connections to the words and their meanings.
- Possible problem-solving strategies/graphic organizers:
    - Consider utilizing the provided graphic organizer for mathematical operations and symbols.
- Possible supports for students who have difficulty justifying their thinking:
    - Ask students to talk to their peers and share ideas to get feedback from their peers.
    - Provide introductory level problems to build student confidence and help them work through how to justify their thinking.
    - Have a class discussion on what it means to justify a solution using an introductory problem.

Task Implementation (After) 15-20 minutes

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during a whole class discussion. Some possible big mathematical ideas to highlight could include:
  - A common misconception
  - Trajectory of sophistication in student ideas (i.e. concrete to abstract; learning trajectories for multiplication or division of decimals)
  - Connection between multiplication and division (could both operations provide the same outcome?)
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
  - How are these strategies alike? How are they different?
  - _____'s strategy is similar to _____'s strategy because _____.
  - How do these connect to our Learning Intentions?
  - Why is this important?
- Highlight student strategies to show the connections, either between different ideas for solutions or to show the connection between levels of sophistication of student ideas (connect strategy of repeated addition to strategy of multiplication – what is similar? Different?). Allow students to ask clarifying questions.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
  - Students can participate in a Gallery Walk to view all strategies prior to coming together to discuss selected strategies.
  - Students can "Think, Pair, Share" strategies for solving.
- Close the lesson by returning to the success criteria. Have students reflect on their progress toward the criteria.

Teacher Reflection About Student Learning

- Additional supports that may further help students with implementation of the task or for students with mathematical difficulties:
  - Students who are at risk of having difficulties should be identified prior to implementing the task and grouped appropriately.
  - Consider modelling expectations of selected components of the task.
  - Provide corrective feedback frequently throughout the task.
  - Frequently relate the math to relevancy in their lives based on individualized student interest.
  - Consider chunking the task into ten-minute intervals using motivational strategies and brain breaks in between.
  - Have students complete the task in chunked amounts of time over the span of two or three days.
- Teacher will use the chart with anticipated student solutions to monitor which students are using which strategies. This will include: possible misconceptions, learning trajectories and sophistication of student ideas, and multiple solution pathways. Next steps based on this information could include:
  - Informing sequence of future tasks. What will come next in instruction to strengthen students' understanding and use of expressions?
  - Informing small groups based on misconceptions that are not addressed in sharing.
  - Use of data obtained from task in order to guide instruction in future tasks.
  - Using an observational tool to assess the level of student engagement on the task.
- After task implementation, the teacher will use the Process Goals rubric to assess student understanding in relation to the process goals. The teacher may decide to focus on one category. Next steps based on this information could include:
  - Informing small groups based on current student engagement with the process goal(s) (i.e. think aloud, using specific sentence frames for communication, etc.).

Planning for Mathematical Discourse

Mathematical Task: Shopping Translations

Content Standard(s): SOL A.1

Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions – Teacher Stays to Hear Response** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions – Teacher Poses Question and Walks Away** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response A: Students may have difficulty with the cost of a computer desk with the expression "two thousand five hundred less than" and may write 2,500 first in the expression	• Ask students to replace the variable with a given value and evaluate the reasonableness of the cost of the computer desk.	• Ask students to replace "two thousand five hundred" and the money they earned this summer with two smaller values that may be more manageable.		
Anticipated Student Response B: Students may have difficulty with the cost of the bike with understanding the word "quantity" and how to represent it algebraically	• Ask students how they would represent the quantity of two small values such as 2 and 5. • Ask students to replace the word quantity with *sum,* or *amount.*	• Ask the student which other words could be used to represent quantity.		
Anticipated Student Response C: Students may struggle with representing the cost of the laptop computer or the cost of a bike because of the fraction involved	• What operation(s) would be used to find half of a value?	• Have students identify a second way they could take a fraction of a value.		

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Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions – Teacher Stays to Hear Response** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions – Teacher Poses Question and Walks Away** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** ● *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* ● *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response D: Students may miscalculate and write an expression incorrectly, resulting in them finding they have enough money to pay for all items	• Can you compare your expressions with a partner?	• Review each value. Are all item costs reasonable?		

Name _____ Date _____

Shopping Translations

You spent your summer working a part-time job in order to be able to purchase the items you need for your first year at college. There are many items you need and there are many items you want. Below is a list of items and their costs expressed verbally. For each item, translate the cost as an algebraic expression, using a variable to represent the amount of money you earned this summer. Then answer the questions shown below the table.

Cost of a laptop computer: one-fourth of the money you earned this summer Translate:	Cost of a computer desk: two thousand five hundred less than the product of the money you earned this summer and two Translate:
Cost of a desk chair: five more than the quotient of the money you earned this summer and twenty Translate:	Cost of a bike: one-eighth of the quantity of two and the money you earned this summer Translate:
Cost of textbooks: twice the amount of money you earned this summer minus three thousand Translate:	Cost of new shoes: square of the quotient of the money you earned this summer and one hundred sixty Translate:
Cost of new clothes: The difference between the money you earned this summer and one thousand five hundred Translate:	Cost of a mini fridge: Three times the square root of the money you earned this summer Translate:

Questions: If you earned $1600.00 this summer at your part time job, will you have enough money to make all the purchases above? Show your work evaluating each expression above. What is the total cost for all your college expenses? If you do not have enough money to cover the expenses, what item(s) would you not purchase? What would your total cost be after eliminating the item(s)?

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	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: ● Uses relationships among mathematical concepts or makes mathematical generalizations	● Demonstrates an understanding of concepts and skills associated with task ● Applies mathematical concepts and skills which lead to a valid and correct solution	● Demonstrates a partial understanding of concepts and skills associated with task ● Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	● Demonstrates no understanding of concepts and skills associated with task ● Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: ● Problem solving strategy is well developed or efficient	● Problem solving strategy displays an understanding of the underlying mathematical concept ● Produces a solution relevant to the problem and confirms the reasonableness of the solution	● Problem solving strategy displays a limited understanding of the underlying mathematical concept ● Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	● A problem-solving strategy is not evident ● Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: ● Reasoning or justification is comprehensive ● Consistently uses precise mathematical language to communicate thinking	● Demonstrates reasoning and/or justifies solution steps ● Supports arguments and claims with evidence ● Uses mathematical language to communicate thinking	● Reasoning or justification of solution steps is limited or contains misconceptions ● Provides limited or inconsistent evidence to support arguments and claims ● Uses limited mathematical language to partially communicate thinking	● Provides no correct reasoning or justification ● Does not provide evidence to support arguments and claims ● Uses no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: ● Uses representations to analyze relationships and extend thinking ● Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	● Uses a representation or multiple representations, with accurate labels, to explore and model the problem ● Makes a mathematical connection that is relevant to the context of the problem	● Uses an incomplete or limited representation to model the problem ● Makes a partial mathematical connection or the connection is not relevant to the context of the problem	● Uses no representation or uses a representation that does not model the problem ● Makes no mathematical connections

Additional Resources/Graphic Organizers/Etc.

Sample Graphic Organizer for Mathematical Operations and Symbols

Addition (+)	Subtraction (-)	Multiplication (x)	Division (÷)