Rich Mathematical Task – Algebra I – *Seesaw Fun*

### Task Overview/Description/Purpose:
- In this task, students will use prerequisite knowledge of solving equations using the representation of a scale balance to discover the difference between equations that result in “no solution” and “infinite solutions.”
- The purpose of this task is to introduce students to the concepts identified above. The goal of this task is for students to discover these concepts before receiving instruction on “how to” solve equations that result in no solution or infinitely many solutions.

### Standards Alignment: Strand – *Equations and Inequalities*

**Primary SOL:**
- A.4 The student will solve
  - a) multistep linear equations in one variable algebraically.

**Related SOL:** 7.12, 8.17

### Learning Intention(s):
- **Content** - I am learning to identify the similarities and differences between equations with no solution and equations with infinitely many solutions.
- **Language** - I am learning to explain my thinking using proper mathematical vocabulary.
- **Social** - I am learning to work with my peers to solve a practical problem.

### Success Criteria (Evidence of Student Learning):
- I can explain what happens to a balance scale representing an equation when either the left side or right side values are different.
- I can explain what happens to a balance scale representing an equation when the left and side values are equal.
- I can write an equation to represent a practical situation.
- I can determine whether or not a solution is exists given a practical situation.

### Mathematics Process Goals

| Problem Solving | • Students will apply mathematical concepts and skills and the relationships among them and choose an appropriate strategy to solve a problem. |
| Communication and Reasoning | • Students will be given unique conditions to the problem and will explain how their conditions affect whether or not a scale can be balanced.  
  • Students will provide work to show how they used their strategy to reach their solution. |
| Connections and Representations | • Students will provide one or more representations of the situation: drawing and/or equation. |
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<table>
<thead>
<tr>
<th>Task Pre-Planning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Approximate Length/Time Frame:</strong> 45 minutes</td>
</tr>
</tbody>
</table>

**Grouping of Students:**
This task is intended to be completed **before** formal instruction on solving equations that result in no solution and infinitely many solutions.

- First have students complete this task independently in order to develop individual thoughts.
- Next, once students have completed the task individually, put them in groups of 3-4 to discuss their findings.
  - Creating heterogeneous groups based on formative assessment
  - Grouping fluent bilingual students with peers who may struggle with English
  - Using student responses to guide your grouping (i.e., don’t put students together who either all answered the same way and don’t put students together who were all incorrect)
  - Using cooperative grouping structures or other protocols to encourage all students in a group to participate

**Materials and Technology:**
- Task printout if completing on paper
- Computer or electronic device (to access activity on Desmos)
- Desmos Activity (make sure you create your own unique class codes for students to complete)
- Paper/pencil or white board/markers (to record their thoughts and sketch models)
- Graphing utility

**Vocabulary:**
- Solution
- No solution
- Infinitely many solutions
- Balanced
- Equal
- Equation

**Anticipate Responses:** See the Planning for Mathematical Discourse Chart (columns 1-3)

<table>
<thead>
<tr>
<th>Task Implementation (Before) 5 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Task Launch:</strong></td>
</tr>
<tr>
<td>- Find a quick video of children playing on a seesaw to hook students. Then ask students about their experiences playing on a seesaw.</td>
</tr>
<tr>
<td>- Ensure students have access to a variety of materials to record their thoughts and develop a problem solving strategy.</td>
</tr>
<tr>
<td>- Share learning intentions and success criteria with students.</td>
</tr>
<tr>
<td>- Access the Desmos Activity provided for students to complete virtually. Be sure to create your own unique class code and share it with students for them to complete.</td>
</tr>
</tbody>
</table>
  - When completing this activity, use the pacing feature to restrict students to complete Part I. |
  - Use live student responses to discuss misconceptions and findings from Part I. |
  - Use the pacing feature to restrict students to complete Part II. |
  - Use live student responses to pause the activity and ask whole group advancing and/or assessing questions. |
  - Use Desmos’s written feedback feature to provide individual students with advancing and/or assessing questions. |
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**Task Implementation (Before)** 5 minutes
- If completed in a virtual setting, use breakout rooms to allow students to work in small groups to discuss their findings.

**Task Implementation (During)** 20-25 minutes

**Directions for Supporting Implementation of the Task**
- **Monitor** – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see the Planning for Mathematical Discourse chart on next page).
- **Select** – Teacher will decide which strategies will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning.
- **Sequence** – Teacher will decide the order in which student ideas will be highlighted (after student task implementation).
- **Connect** – Teacher will consider ways to facilitate connections between different student responses.

**Suggestions for Additional Student Support**
- May include, but not limited to:
  - Possible use of sentences frames to support student thinking
    - I think _____ because ...
    - I believe that ________ will happen because ...
    - This reminds me of ...
    - Another idea I had was...
    - I was confused (wondering) about...
    - I believe that is true because ...
    - How or why did you...?
    - I agree (disagree) because...
    - Your answer/strategy reminds me of...
    - Can you explain more about...?
    - I would like to add on...
  - Provide oral instructions
  - Allow students to provide oral explanations
  - Some students may struggle visualizing students with backpacks on the seesaw on Part II. Consider providing them a visual to see what is happening.

**Task Implementation (After)** 10-15 minutes

**Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:**
- Based on the actual student responses, sequence and select particular students to present their mathematical work during a whole class discussion. Consider students who have plausible methods, but create equations using common misconceptions. These misconceptions can include, but are not limited to:
  - Assuming that having equally weighted backpacks means the seesaw is balanced; the student does not recognize that the different weights of the friends affects the balance.
  - When finding a backpack weight to try balancing the seesaw, giving an actual weight and not keeping this in terms of \( b \).
  - Not creating an equations where there are variables on both sides – only putting the variable on one side.
- Connect different students’ responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
  - How are these strategies alike? How are they different?
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- ________’s strategy is similar to ________’s strategy because ________.
- How do these connect to our Learning Intentions?
- Why is this important?
- Highlight student strategies either to show the connections between different ideas for solutions or to show the connection between levels of sophistication of student ideas. Allow students to ask clarifying questions.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
  - Students can participate in a Gallery Walk to view all strategies prior to coming together to discuss selected strategies.
  - Students can “Think, Pair, Share” strategies for solving.
- Close the lesson by returning to the success criteria. Have students reflect on their progress toward the criteria.

### Teacher Reflection About Student Learning

- Were the learning intentions and success criteria met? By all students? Most students? Some students?
- Were students able to explain and justify their thinking?
- What common misconceptions arose?
- What variety of strategies did students use to make sense of the task?
- What was the level of student engagement during the task?
- Are there additional supports that may have further helped students with implementation of the task?
- What common errors/misconceptions did students have that were not expected?
- Because of completing this task, which process goals might need additional attention?
- Are students able to communicate their ideas both verbally and in writing?
- Are students able to explain the difference between an equation that results in no solution vs. an equation that results in infinitely many solutions?
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Planning for Mathematical Discourse

Mathematical Task: *Seesaw Fun*  
Content Standard(s): SOL A.4a

<table>
<thead>
<tr>
<th>Teacher Completes Prior to Task Implementation</th>
<th>Teacher Completes During Task Implementation</th>
</tr>
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</table>
| **Anticipated Student Response/Strategy** | **Assessing Questions – Teacher Stays to Hear Response**  
Provide examples of possible correct student responses along with examples of student errors/misconceptions  
Teacher questioning that allows student to explain and clarify thinking | **Advancing Questions – Teacher Poses Question and Walks Away**  
Teacher questioning that moves thinking forward | **List of Students Providing Response**  
Who? Which students used this strategy? |
| **Anticipated Student Response A:**  
In Part I student is unable to correctly explain/model what happens when different friends sit on the seesaw AND in Part II, student is unable to create equations to model the two different situations. | **Can you identify which friends you chose to sit on the seesaw and why?**  
**What equations did you write in Part II?**  
**What evidence do you have to support the equations that you wrote?**  
**What is the difference between an expression and an equation?**  
**How do the weights affect what happens on the seesaw?**  
**What is the expression you wrote for the heavier student while wearing their backpack?** | **Based on that equation, how much weight is their friend missing that keeps the seesaw from being balanced?**  
**How can you give that missing weight to other friend so the seesaw is balanced?**  
**How do you know your solution makes sense?** | **Discussion Order - sequencing student responses**  
Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion  
Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion |

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<tr>
<th>Teacher Completes Prior to Task Implementation</th>
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<th>List of Students Providing Response</th>
<th>Discussion Order - sequencing student responses</th>
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<tr>
<td><strong>Anticipated Student Response/Strategy</strong></td>
<td>Provide examples of possible correct student responses along with examples of student errors/misconceptions</td>
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<td>Who? Which students used this strategy?</td>
<td>Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</td>
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#### Anticipated Student Response B:
In Part I, student is able to correctly explain/model what happens when different friends sit on the seesaw, **BUT** in Part II, student is unable to create equations to model the two situations.

- What equations did you write in Part II?
- What is the expression you wrote for the heavier student while wearing their backpack?
- What evidence do you have to support the equations that you wrote?
- What is the difference between an expression and an equation?
- Based on that equation, how much weight is their friend missing that keeps the seesaw from being balanced?
- How can you give that missing weight to other friend so the seesaw is balanced?
- Is it possible for the seesaw to be balanced?
- How do you know your solution makes sense?

#### Anticipated Student Response C:
In Part I, student is able to correctly explain/model what happens when different friends sit on the seesaw, **BUT** in Part II, the student is able to model and explain why the equation has no solution, but is not able to write an expression in terms of \( b \) to balance the seesaw.

- What equations did you write in Part II?
- What evidence do you have to support the equations that you wrote?
- What is the difference between an expression and an equation?
- What is the expression you wrote for the
- Based on that equation, how much weight is their friend missing that keeps the seesaw from being balanced?
- How can you give that missing weight to other friend so the seesaw is balanced?
- How do you know your solution makes sense?
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<td>Teacher questioning that moves thinking forward</td>
<td></td>
<td>♦ Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</td>
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<td>Provide examples of possible correct student responses along with examples of student errors/misconceptions</td>
<td></td>
<td></td>
<td></td>
<td>♦ Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</td>
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<td></td>
<td>heavier student while wearing their backpack?</td>
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<tr>
<td><strong>Anticipated Student Response D:</strong></td>
<td>• What equations did you write in Part II?</td>
<td>• If “b” has to be the same, would it be possible for the seesaw to ever balance? Why do you think that?</td>
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<tr>
<td>In Part I, student is able to correctly explain/model what happens when different friends sit on the seesaw, <strong>BUT</strong> in Part II, the student is able write an expression in terms of b to balance the scale, but is unable to write an equation that represents the situation that results in no solution.</td>
<td>• What evidence do you have to support the equations that you wrote?</td>
<td>• How do you know your solution makes sense?</td>
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<td></td>
<td>• What is the difference between an expression and an equation?</td>
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<td><strong>Anticipated Student Response E:</strong></td>
<td>• What equations did you write in Part II?</td>
<td>• What do you notice about the left side of each equation and the right side of each equation?</td>
<td></td>
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</tr>
<tr>
<td>In Part I, student is able to correctly explain/model what happens when different friends sit on the seesaw, <strong>AND</strong> in Part II, the student is able to write/explain an equation when there is no solution and is able to write/explain an equation that balances the seesaw, <strong>BUT</strong> does not recognize the differences in</td>
<td>• What evidence do you have to support the equations that you wrote?</td>
<td>• How do you know your solutions make sense?</td>
<td></td>
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<tr>
<td></td>
<td>• What were your two equations?</td>
<td>• Do both of these models represent a true equation?</td>
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<tr>
<td><strong>Anticipated Student Response F:</strong></td>
<td><strong>Advancing Questions – Teacher Poses Question and Walks Away</strong></td>
</tr>
<tr>
<td>In Part I, student is able to correctly explain/model what happens when different friends sit on the seesaw, <strong>AND</strong> in Part II, the student is able to write/explain an equation when there is no solution and is able to write/explain an equation that balances the seesaw, <strong>AND</strong> recognizes the differences in the two equations written in Part II.</td>
<td><em>Teacher questioning that moves thinking forward</em></td>
</tr>
<tr>
<td>• What equations did you write in Part II?</td>
<td><strong>List of Students Providing Response</strong></td>
</tr>
<tr>
<td>• What evidence do you have to support the equations that you wrote?</td>
<td><em>Who? Which students used this strategy?</em></td>
</tr>
<tr>
<td>• How would you solve these equations?</td>
<td><strong>Discussion Order - sequencing student responses</strong></td>
</tr>
<tr>
<td>• What do you notice about the solution to the equations?</td>
<td>• Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion</td>
</tr>
<tr>
<td>• Do both of these models represent a true equation?</td>
<td>• Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion</td>
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the two equations written in Part II.
Seesaw Fun

Seesaws (pictured above) are a common item children play on at playgrounds. Juan, Charisse, Mark, and Reniyah are at their playground and want to play on this.

Part I:

a. The Desmos activity randomly assigned weights for each of the four friends. Record their weights in the spaces below.

Juan = ______ pounds
Charisse = ______ pounds
Mark = ______ pounds
Reniyah = ______ pounds

b. Using the weights listed above, answer the following questions if only Juan is sitting on the left side of the seesaw.

1. What would happen to the seesaw if only Charisse sat on the right side? Why do you think that?

2. What would happen to the seesaw if only Mark sat on the right side? Why do you think that?

3. What would happen to the seesaw if only Reniyah sat on the right side? Why do you think that?
Rich Mathematical Task – Algebra I – Seesaw Fun

Part II:

a. Choose two friends with different weights to sit on the seesaw and record their names below. Be sure to place the heavier friend on the left side and ensure the two friends have different weights:
   1. Left side: __________________
   2. Right side: __________________

b. These two friends are both wearing backpacks. If the weights of the backpacks of each friend are the same, is it possible for the seesaw to be balanced?
   1. Explain your thinking.
   2. Let b = the weight of the backpack of the heavier friend. Write an algebraic equation that represents the situation.

c. Given that the heavier friend is still sitting on the left side and “b” still represents the weight of their backpack:
   1. If the weights of the backpacks are no longer the same, write an expression for the weight of the backpack of the friend sitting on the right side (in terms of b) so the seesaw will balance.
   2. Now that the seesaw is balanced, write an algebraic equation (in terms of b) that represents the situation.

d. How are the equations in part b and c alike or different?

e. What affect does the weight of the backpack have on whether or not the seesaw is balanced?
Rich Mathematical Task Rubric

### Mathematical Understanding

<table>
<thead>
<tr>
<th>Level</th>
<th>Proficient Plus:</th>
<th>Proficient</th>
<th>Developing</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced</td>
<td>Uses relationships among mathematical concepts or makes mathematical generalizations</td>
<td>Demonstrates an understanding of concepts and skills associated with task</td>
<td>Demonstrates a partial understanding of concepts and skills associated with task</td>
<td>Demonstrates no understanding of concepts and skills associated with task</td>
</tr>
<tr>
<td>Proficient</td>
<td>Problem solving strategy displays an understanding of the underlying mathematical concept</td>
<td>Problem solving strategy displays a limited understanding of the underlying mathematical concept</td>
<td>Problem solving strategy displays a limited understanding of the underlying mathematical concept</td>
<td>A problem solving strategy is not evident</td>
</tr>
<tr>
<td>Developing</td>
<td>Produces a solution relevant to the problem and confirms the reasonableness of the solution</td>
<td>Produces a solution relevant to the problem but does not confirm the reasonableness of the solution</td>
<td>Reasoning or justification of solution steps is limited or contains misconceptions</td>
<td>Provides no correct reasoning or justification</td>
</tr>
<tr>
<td>Emerging</td>
<td>Makes a mathematical connection that is relevant to the context of the problem</td>
<td>Uses a representation or multiple representations, with accurate labels, to explore and model the problem</td>
<td>Uses an incomplete or limited representation to model the problem</td>
<td>Uses no representation or uses a representation that does not model the problem</td>
</tr>
</tbody>
</table>

### Problem Solving

<table>
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<tr>
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<th>Proficient</th>
<th>Developing</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Advanced</td>
<td>Problem solving strategy is well developed or efficient</td>
<td>Problem solving strategy displays an understanding of the underlying mathematical concept</td>
<td>Produces a solution relevant to the problem and confirms the reasonableness of the solution</td>
<td>A problem solving strategy is not evident</td>
</tr>
<tr>
<td>Proficient</td>
<td>Problem solving strategy displays an understanding of the underlying mathematical concept</td>
<td>Problem solving strategy displays a limited understanding of the underlying mathematical concept</td>
<td>Produces a solution relevant to the problem but does not confirm the reasonableness of the solution</td>
<td>Does not produce a solution that is relevant to the problem</td>
</tr>
<tr>
<td>Developing</td>
<td>Produces a solution relevant to the problem but does not confirm the reasonableness of the solution</td>
<td>Produces a solution relevant to the problem but does not confirm the reasonableness of the solution</td>
<td>Does not produce a solution that is relevant to the problem</td>
<td></td>
</tr>
<tr>
<td>Emerging</td>
<td>Problem solving strategy is not evident</td>
<td>Problem solving strategy is not evident</td>
<td>Does not produce a solution that is relevant to the problem</td>
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### Communication and Reasoning

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<th>Proficient</th>
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<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced</td>
<td>Reasoning or justification is comprehensive</td>
<td>Demonstrates reasoning and/or justifies solution steps</td>
<td>Reasoning or justification of solution steps is limited or contains misconceptions</td>
<td>Provides no correct reasoning or justification</td>
</tr>
<tr>
<td>Proficient</td>
<td>Consistently uses precise mathematical language to communicate thinking</td>
<td>Supports arguments and claims with evidence</td>
<td>Provides limited or inconsistent evidence to support arguments and claims</td>
<td>Does not provide evidence to support arguments and claims</td>
</tr>
<tr>
<td>Developing</td>
<td>Uses mathematical language to communicate thinking</td>
<td>Uses mathematical language to communicate thinking</td>
<td>Uses limited mathematical language to partially communicate thinking</td>
<td>Uses no mathematical language to communicate thinking</td>
</tr>
<tr>
<td>Emerging</td>
<td>Provides no correct reasoning or justification</td>
<td>Provides no correct reasoning or justification</td>
<td>Provides limited or inconsistent evidence to support arguments and claims</td>
<td>Provides no correct reasoning or justification</td>
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### Representations and Connections

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<td>Advanced</td>
<td>Uses representations to analyze relationships and extend thinking</td>
<td>Uses a representation or multiple representations, with accurate labels, to explore and model the problem</td>
<td>Uses an incomplete or limited representation to model the problem</td>
<td>Uses no representation or uses a representation that does not model the problem</td>
</tr>
<tr>
<td>Proficient</td>
<td>Uses mathematical connections to extend the solution to other mathematics or to deepen understanding</td>
<td>Makes a mathematical connection that is relevant to the context of the problem</td>
<td>Makes a partial mathematical connection or the connection is not relevant to the context of the problem</td>
<td>Makes no mathematical connections</td>
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<tr>
<td>Developing</td>
<td>Provides no correct reasoning or justification</td>
<td>Provides no correct reasoning or justification</td>
<td>Provides limited or inconsistent evidence to support arguments and claims</td>
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<td>Emerging</td>
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a. Choose two friends with different weights to sit on the seesaw and record their names below. Be sure to place the heavier friend on the left side and ensure the two friends have different weights:
   1. Left side: ________________
   2. Right side: ________________

b. These two friends are both wearing backpacks. If the weights of the backpacks of each friend are the same, is it possible for the seesaw to be balanced?
   1. Explain your thinking.
   2. Let b = the weight of the backpack of the heavier friend. Write an algebraic equation that represents the situation.

c. Given that the heavier friend is still sitting on the left side and “b” still represents the weight of their backpack:
   1. If the weights of the backpacks are no longer the same, write an expression for the weight of the backpack of the friend sitting on the right side (in terms of b) so the seesaw will balance.
   2. Now that the seesaw is balanced, write an algebraic equation (in terms of b) that represents the situation.

d. How are the equations in part b and c alike or different?

e. What affect does the weight of the backpack have on whether or not the seesaw is balanced?