

Rich Mathematical Task – Geometry – *Logical Drivers*

Task Overview/Description/Purpose:	
<ul style="list-style-type: none"> In this task, students will use the Virginia Driver’s Manual and review the penalties, section 5. Students will learn about the driving laws in Virginia and the consequences for violating those laws. Then, they will create a series of logical arguments to understand how to think logically through the driving laws and judge the validity of those arguments. The purpose of this task is for students to use deductive reasoning to determine the validity of a logical argument. They will identify the converse, inverse, and contrapositive of a conditional statement and translate short verbal arguments into symbolic form. 	
Standards Alignment: Strand – <i>Computation and Estimation</i>	
<p>Primary SOL: G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include</p> <ol style="list-style-type: none"> identifying the converse, inverse, and contrapositive of a conditional statement; translating a short verbal argument into symbolic form; and determining the validity of a logical argument. 	
Learning Intention(s):	
<ul style="list-style-type: none"> Content - I am learning how to write the converse, inverse, and contrapositive of a conditional statement. I am learning how to translate a short verbal argument into symbolic form and determine the validity of a logical argument. Language - I am learning to explain and justify my thinking and reasoning when determining the validity of a logical argument. Social - I am learning how to communicate my mathematical thinking to my peers and ask probing questions that help my peers and me advance our thinking about logical arguments. 	
Success Criteria (Evidence of Student Learning):	
<ul style="list-style-type: none"> I can write and determine the validity of the converse, inverse, and contrapositive of a conditional statement. I can represent logical arguments using logic symbols. I can explain my thinking using the laws of syllogism, contrapositive, and detachment. I can logically communicate when an argument is false by providing a counterexample. 	
Mathematics Process Goals	
Problem Solving	<ul style="list-style-type: none"> Students will create a series of conditional statements and logic laws using the laws and penalties in the Virginia Driver’s Manual and determine the validity of those conditional statements. Students will provide a counterexample to the converse of their conditional statement to justify when the converse is false.
Communication and Reasoning	<ul style="list-style-type: none"> Students will support arguments and claims with evidence when determining the validity of a logical argument or the validity of the converse, inverse, or contrapositive of a conditional statement. Students will use mathematical language to justify their reasoning.
Connections and Representations	<ul style="list-style-type: none"> Students will use logic symbols to represent conditional statements, the converse, inverse, and the contrapositive of the conditional statements. Students will make connection that are relevant to the context of the problems using deductive reasoning.

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Task Pre-Planning

Approximate Length/Time Frame: 90 minutes

Grouping of Students:

Students can be grouped based on language or social needs. Consider using a formative assessment to group students for the task based on their understanding of prerequisite knowledge. When completed with the task, ask students to share their solutions with another individual or team to determine the validity of their task. Finally, create small groups of students to share and discuss problems 4 and 7 in order for students to identify commonalities in responses.

Materials and Technology:

- Copy of task
- [Virginia Driver's Manual Section 5, penalties \(p.27-30\)](#)
- Pencils or electronic platform for typing and submitting work

Vocabulary:

- deductive reasoning
- law of syllogism
- law of detachment
- law of contrapositive
- counterexample
- conditional Statement
- converse
- inverse
- contrapositive
- biconditional
- hypothesis
- conclusion
- inductive reasoning

Anticipate Responses: See the Planning for Mathematical Discourse Chart (columns 1-3).

Task Implementation (Before) 20 minutes

Task Launch:

- Consider using the grouping suggestions to place students in appropriate groups based on their needs. Assign groups before showing the introduction video and introducing the task.
- Teachers should share their learning intentions and success criteria prior to starting the task.
- Teachers may show [this introduction video](#) before the task in order to hook students and introduce a logical argument using conditional statement related to the penalties for driving under the influence. Have students read and answer the following questions.
 - What are some penalties you know of for driving under the influence?
 - Can you create a conditional statement based on what you know about driving under the influence in Virginia and the consequence for breaking that law?
 - What are some other driving laws you know of and what are some penalties for violating those laws?
- Students will use underlining, highlighting, cue words, or a visual vocabulary word wall to help make sense of the task.
- Students will access prior knowledge and vocabulary regarding the properties of polygons using the VDOE Word Wall Cards. Have students create a Frayer model for words they have not mastered.

Task Implementation (During) 30 minutes

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Directions for Supporting Implementation of the Task

- Monitor – Teacher will listen and observe students as they work on task and ask assessing or advancing questions (see the Planning for Mathematical Discourse chart on next page).
- Select – Teacher will decide which strategies will be highlighted (after student task implementation) that will advance mathematical ideas and support student learning.
- Sequence – Teacher will decide the order in which student ideas will be highlighted (after student task implementation).
- Connect – Teacher will consider ways to facilitate connections between different student responses.
 - Students work in purposefully planned groups for 20-25 minutes to explore strategies, share ideas and transfer their ideas to paper using pictures, words, and symbols.
 - As the teacher is monitoring, teacher will look for strategies that are being used and record on Planning Chart.
 - The teacher should use questions to assess or advance student thinking.
 - Students should be encouraged to explore different strategies for solving and evaluate effectiveness.

Suggestions For Additional Student Support

- Possible use of sentences frames to support student thinking
 - The converse of my conditional statement is false because ...
 - I chose _____ as a counterexample to my converse because ...
 - I cannot write a biconditional of my conditional statement because ...
- Possible actions to support students who have difficulty justifying their thinking
 - Allow students to choose a rule or law that is not related to driving laws if they are having difficulty understanding the truth value of their statements. Are there any patterns in the converse, inverse, and contrapositive statements that could support their driving law statements?
 - Consider introducing a truth table to judge the validity of students' statements.
 - Ask students to talk through their thought process with another student before writing anything on their paper.
- Possible actions to support vocabulary development
 - Have students complete a Frayer model.
 - Display VDOE Word Wall Cards.
 - Make word associations clear, e.g. focus on BI in biconditional meaning "two."
 - Ensure when students are speaking/writing that they are utilizing the proper vocabulary terms
 - Have students create a math vocabulary book.
 - Ask students to speak with each other about what they know about logic laws, conditional statements, and deductive reasoning.
- Possible problem solving strategies/graphic organizers
 - Consider providing examples of conditional statements and counterexamples to the converse, inverse, and contrapositive of the conditional statements.

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Task Implementation (After) 20-30 minutes

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during a whole class discussion. Some possible big mathematical ideas to highlight could include:
 - A common misconception
 - Trajectory of sophistication in student ideas (i.e. concrete to abstract; learning trajectories for multiplication or division of decimals)
 - Connection between multiplication and division (could both operations provide the same outcome?)
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task. Possible questions and sentence frames to connect student strategies:
 - How are these strategies alike? How are they different?
 - _____'s strategy is similar to _____'s strategy because _____
 - How do these connect to our Learning Intentions?
 - Why is this important?
- Highlight student strategies to show the connections, either between different ideas for solutions or to show the connection between levels of sophistication of student ideas (connect strategy of repeated addition to strategy of multiplication – what is similar? Different?). Allow students to ask clarifying questions.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion.
 - Students can participate in a Gallery Walk to view all strategies prior to coming together to discuss selected strategies.
 - Students can “Think, Pair, Share” strategies for solving.
- Close the lesson by returning to the success criteria. Have students reflect on their progress toward the criteria.

Teacher Reflection About Student Learning

- Teacher will use the chart with anticipated student solutions to monitor which students are using which strategies. This will include: possible misconceptions, learning trajectories and sophistication of student ideas, and multiple solution pathways. Next steps based on this information could include:
 - Informing sequence of future tasks. What will come next in instruction to further student thinking in logic?
 - Informing small groups based on misconceptions that are not addressed in sharing.
- After task implementation, the teacher will use the Process Goals rubric to assess student understanding in relation to the process goals. The teacher may decide to focus on one category. Next steps based on this information could include:
 - Informing small groups based on current student engagement with the process goal(s) (i.e. think aloud, using specific sentence frames for communication, etc.).

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Planning for Mathematical Discourse

Mathematical Task: Logical Drivers

Content Standard(s): SOL G.1

Teacher Completes Prior to Task Implementation			Teacher Completes During Task Implementation	
Anticipated Student Response/Strategy <i>Provide examples of possible correct student responses along with examples of student errors/misconceptions</i>	Assessing Questions – Teacher Stays to Hear Response <i>Teacher questioning that allows student to explain and clarify thinking</i>	Advancing Questions – Teacher Poses Question and Walks Away <i>Teacher questioning that moves thinking forward</i>	List of Students Providing Response <i>Who? Which students used this strategy?</i>	Discussion Order - sequencing student responses <ul style="list-style-type: none"> Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion
Anticipated Student Response A: Students may have difficulty narrowing down the laws and penalties to one conditional statement.	<ul style="list-style-type: none"> Can you choose one paragraph of the penalties section and try to create a conditional statement using information from that one paragraph? 	<ul style="list-style-type: none"> What are some driving laws and consequences you already know before reading the manual? 		
Anticipated Student Response B: Students may not understand how to represent the hypothesis and conclusion using variables and logic symbols.	<ul style="list-style-type: none"> What are the parts of a conditional statement? 	<ul style="list-style-type: none"> How can you rewrite a conditional statement only using symbols and variables? 		
Anticipated Student Response C: Students may be unable to determine if the inverse, converse, or contrapositive of their conditional statement is false.	<ul style="list-style-type: none"> Is the conclusion true or false with the given hypothesis? What is a counterexample? 	<ul style="list-style-type: none"> Can you create a counterexample to the statement that could help prove the statement is false? 		
Anticipated Student Response D: Students may have difficulty narrowing down the laws and penalties to write their own law	<ul style="list-style-type: none"> Can you choose one paragraph of the penalties section and try to create 	<ul style="list-style-type: none"> What are some driving laws and consequences you already know before reading the manual? 		

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of syllogism, law of detachment, or law of contrapositive.	the law using information from that one paragraph? <ul style="list-style-type: none"> ● Can you build off of your conditional statement you originally wrote in (1)? ● What are the law of syllogism, law of detachment and law of contrapositive? 			

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Name _____

Date _____

Logical Drivers

Upon receiving your driver’s license, you become responsible for following many traffic laws that keep yourself and others safe while on the road. If you violate these traffic laws you could be ticketed, fined, and face many other penalties. A list of the traffic laws in Virginia and their penalties if violated, can be found in Virginia’s Driver Manual.

1. Review [Virginia Driver's Manual Section 5 \(Pages 27-30\)](#) and identify one law and the consequences for violating that law. Write a conditional statement where the hypothesis contains the law that is being broken, and the conclusion contains the penalty for breaking that law.
2. Define variables for your hypothesis and conclusion and rewrite your conditional statement using your variables and logic symbols.
3. Write the converse, inverse, and contrapositive of your conditional statement. Rewrite each statement in symbolic form.

Statement	Symbolic Form
Converse -	
Inverse -	
Contrapositive -	

4. Is the converse of your conditional statement true? If not, provide a counterexample.

5. Is the inverse of your conditional statement true? If not, provide a counterexample.

6. Is the contrapositive of you conditional statement true? If not, provide a counterexample.

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7. Is it possible to write a biconditional for your conditional statement? If not, explain why.
8. Often times there are a series of consequences that occur when a policy is not followed. For example:
- If you are tardy to class more than three times in a quarter, then you will receive a detention. If you receive a detention, you will be unable to attend after school activities for a day. Therefore, if you are tardy to class more than three times in a quarter, then you will be unable to attend after school activities for a day.
- This series of consequences represents the law of syllogism.
- Review the driving laws and consequences again. Create conditional statements that follow the law of syllogism and provide the final conclusion like in the example provided above.
9. Use logic symbols to represent the law of syllogism.
10. Represent a violation of a driving law and its consequence using the law of detachment. Provide a valid conclusion.
11. Represent a violation of a driving law and its consequence using the law of contrapositives. Provide a valid conclusion.

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Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	<p>Proficient Plus:</p> <ul style="list-style-type: none"> • Uses relationships among mathematical concepts or makes mathematical generalizations 	<ul style="list-style-type: none"> • Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution 	<ul style="list-style-type: none"> • Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution 	<ul style="list-style-type: none"> • Demonstrates no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	<p>Proficient Plus:</p> <ul style="list-style-type: none"> • Problem solving strategy is well developed or efficient 	<ul style="list-style-type: none"> • Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution 	<ul style="list-style-type: none"> • Problem solving strategy displays a limited understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution 	<ul style="list-style-type: none"> • A problem solving strategy is not evident • Does not produce a solution that is relevant to the problem
Communication and Reasoning	<p>Proficient Plus:</p> <ul style="list-style-type: none"> • Reasoning or justification is comprehensive • Consistently uses precise mathematical language to communicate thinking 	<ul style="list-style-type: none"> • Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to communicate thinking 	<ul style="list-style-type: none"> • Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking 	<ul style="list-style-type: none"> • Provides no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses no mathematical language to communicate thinking
Representations and Connections	<p>Proficient Plus:</p> <ul style="list-style-type: none"> • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding 	<ul style="list-style-type: none"> • Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem 	<ul style="list-style-type: none"> • Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections