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Mathematics Standards of Learning

for Virginia Public Schools

Board of Education
Commonwealth of Virginia

February ~~2009~~2016

Mathematics Standards of Learning

for Virginia Public Schools

Adopted in ~~February 2009~~September 2016
by the Board of Education

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Preface

~~The Standards of Learning in this publication represent a significant development in public education in Virginia. These standards focus on the mathematical knowledge and skills all students need for the future, and they have been aligned with national expectations for postsecondary success.~~

In 1995, the Virginia Board of Education published Standards of Learning in English, mathematics, science, and history and social science for kindergarten through grade 12. Subsequently, Standards of Learning were developed for all academic content areas. The Standards of Learning provide a framework for instructional programs designed to raise the academic achievement of all students in Virginia and ~~are an important part of Virginia's efforts to provide challenging educational programs in the public schools~~prepare them for college and careers.

~~The Standards of Learning set reasonable targets and expectations for what teachers need to teach and students need to learn. The standards are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught; the standards are to be incorporated into a broader, locally designed curriculum. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.~~

The Standards of Learning are recognized as a model for other states. Pursuant to legislation from the 2000 Virginia General Assembly, the Board of Education established a seven-year cycle for review of the Standards of Learning. As a result, the 1995 *Mathematics Standards of Learning* were reviewed in 2001, 2009, and 2016, the results of which are contained in this document. The standards~~They~~ were ~~develop~~revised ~~through a series of public hearings and the efforts of~~with input from parents, teachers, administrators, representatives from higher education, and the business ~~and industry leaders~~community. The standards set clear, concise, and measurable academic expectations for ~~young people~~students. Parents and guardians are encouraged to work with their children, their children's teachers, and their children's schools to help them achieve these academic standards.

~~A major objective of Virginia's educational agenda is to give the citizens of Virginia a program of public education that is among the best in the nation and that meets the needs of all young people~~students. ~~in Virginia. These Standards of Learning chart the course for achieving that objective.~~

Introduction

Students today require more rigorous mathematical knowledge and skills to pursue higher education, to compete in a global workforce, and to be informed citizens. Students must gain an understanding of fundamental ideas in number sense, computation, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and they must develop proficiency in mathematical skills.

The 2016 *Mathematics Standards of Learning* ~~for mathematics~~ identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. ~~Recommendations and reports from Achieve,~~Information from the College Board, ~~and~~ ACT, ~~as well as~~ the National Assessment of Educational Progress (NAEP) Framework~~s~~, the *Curriculum Focal Points* from the National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics* from NCTM, *Focus in High School Mathematics: Reasoning and Sense Making* from NCTM, ~~the Singapore Curricula,~~ the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report from the American Statistical Association, and the Report of the President's National Mathematics Advisory Panel were considered in identifying mathematics content necessary for success for all students in postsecondary pursuits.

Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands ~~or topics~~ are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement~~;~~ and Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning ~~for~~ within each strand progress in complexity ~~at each~~throughout the grade levels and ~~throughout the~~into high school course~~s~~ content. While the standards are organized by strand and identified numerically, local curricula and pacing guides should determine the instructional sequence of the content.

The 2016 *Mathematics Standards of Learning Curriculum Framework*, ~~is~~ a companion document to the 2016 *Mathematics Standards of Learning*, ~~that~~ amplifies the ~~Mathematics Standards of Learning~~standards and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and Curriculum Framework into a broader, locally-designed curriculum. The Curriculum Framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students. ~~provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn.~~

~~The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.~~

Mathematical Process Goals for Students

~~Students today require more rigorous mathematical knowledge and skills to pursue higher education, to compete in a technologically sophisticated work force, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and they must develop proficiency in mathematical skills.~~ [Moved to Introduction] ~~In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs. Hence, the use of technology must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skills should be the shared responsibility of teachers of all disciplines.~~[Moved to Role of Technology section]

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real ~~life~~ world data and situations within and outside mathematics and then apply appropriate strategies to ~~find~~ determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become ~~competent~~ mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning usin~~g~~e the language of mathematics, including specialized vocabulary and symbolic~~s~~ notation, to express mathematical ideas with ~~precisely~~precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will ~~learn to~~ use number sense to apply proportional and spatial reasoning and to reason from a variety of representations ~~such as graphs, tables, and charts~~.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics ~~to one another~~ and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections ~~between~~ among different areas of mathematics and between mathematics and other disciplines, ~~especially science~~and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships ~~with~~ using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections ~~move easily~~ among different representations – physical, visual, symbolic, ~~graphical, numerical, algebraic,~~ verbal, and contextual~~physical~~ – and recognize that representation is both a process and a product.

The Role of Instructional Technology

Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, computing devices, and other forms of technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs. The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation.

Computational Fluency

Mathematics instruction must simultaneously develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning. Concurrent development of conceptual understanding and computational skills can enhance student's problem-solving skills.

Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of fourth grade. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. While preparing students for the study of Algebra I, the Mathematics Standards of Learning develop statistical and geometric concepts that prepare students for the study of courses like Statistics, Geometry, and other higher-level content. "Algebra readiness" describes students' mastery of content that adequately prepares them for the study of content outlined in the courses above the level of kindergarten through grade 8 mathematics. The determination of "algebra readiness" should be based on the mastery of the mathematics content and the ability to apply the Mathematics Standards of Learning for kindergarten through grade 8.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Mathematics instruction that requires students to think critically, to reason, to develop strategies for problem solving, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. Mathematics instruction should address the individual needs of all learners, including students with disabilities, gifted learners, and English language learners.

Kindergarten

The kindergarten standards place emphasis on developing the concept of number by counting; combining, sorting, and comparing sets of objects; recognizing and describing simple repeating patterns; and recognizing shapes and sizes of figures and objects. Students will investigate ~~nonstandard~~ measurement through direct comparisons, collect data, and create graphs. The ~~idea~~concept of fractions ~~will be~~is introduced through sharing experiences.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations.

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.~~

Number and Number Sense

~~Focus: Whole Number Concepts~~

~~K.1 The student, given two sets, each containing 10 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence.~~ [Moved to K.2]

K.~~1~~2 The student~~, given a set containing 15 or fewer concrete objects,~~ will

a) tell how many are in ~~the~~ a given set of 20 or fewer objects by counting ~~the number of objects~~ orally; and

b) read, write, and represent numbers from 0 through 20. ~~the numeral to tell how many are in the set; and~~

~~c) select the corresponding numeral from a given set of numerals.~~ [Moved to EKS]

K.~~2~~1 The student, given no more than three ~~two~~ sets, each set containing 10 or fewer concrete objects, will

a) ~~identify~~compare and describe one set as having more, fewer, or the same number of ~~members~~ objects as the other set(s)~~,~~; and ~~using the concept of one-to-one correspondence.~~

b) compare and order sets from least to greatest and greatest to least.

K.3 The student will~~, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object.~~ [Ordinal positions moved to 1.3]

a) recognize and describe with fluency part-whole relationships for numbers up to 5; and

b) investigate and describe part-whole relationships for numbers up to 10.

K.4 The student will
a) count forward to 100 starting at any number between 0 and 100;
b) count ~~and~~ backward by ones when given any number between 1 and 10~~from 10~~; and
c) identify ~~one more than a number and one less than a number; and~~the number after and the number before when given any number between 0 and100~~count by fives and tens to 100~~. [Included in 1.2]

K.5 The student will ~~identify the parts of a set and/or region that represent fractions for halves and fourths.~~ investigate fractions by solving practical problems involving equal sharing with two or four sharers.

Computation and Estimation

~~Focus: Whole Number Operations~~

K.6 The student will model and solve single-step story and picture problems with sums to 10 and differences within 10~~adding and subtracting whole numbers~~, using ~~up to 10~~ concrete objects.

Measurement and Geometry

~~Focus: Instruments and Attributes~~

K.7 The student will recognize the attributes of a penny, nickel, dime, and quarter and identify the value of each coin. ~~and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.~~ [Value of a collection included in 1.7]

~~K.8~~ ~~The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).~~ [Moved each instrument to the standard where the content is first taught (ruler – SOL 2.8; scale – SOL 2.8; clock – SOL 1.9; thermometer – SOL 2.11]

K.8~~9~~ The student will investigate the passage of time by reading and interpreting a calendar. [Moved from 1.11] ~~tell time to the hour, using analog and digital clocks.~~ [Time to hour moved to 1.9a

K.9~~10~~ The student will compare two objects or events, using direct comparisons ~~or nonstandard units of measure~~, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder), volume (more, less), and time (longer, shorter). ~~Examples of nonstandard units include foot length, hand span, new pencil, paper clip, and block.~~ [Nonstandard units included in 1.10]

~~Geometry~~

~~*Focus:* Plane Figures~~

K.10~~11~~ The student will
a) identify~~,~~ and describe~~, and trace~~ plane ~~geometric~~ figures (circle, triangle, square, and rectangle); ~~and~~
b) compare the size (smaller, larger~~, smaller~~) and shape of plane ~~geometric~~ figures (circle, triangle, square, and rectangle)~~.~~; and
c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space. [Moved from K.12]

~~K.12~~ ~~The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.~~ [Moved to K.10c]

Probability and Statistics

~~Focus: Data Collection and Display~~

K.11~~13~~ The student will ~~gather data by counting and tallying.~~ [Tallying included in 1.14a]
a) collect, organize, and represent data; and
b) read and interpret data in object graphs, pictures graphs, and tables. [Moved from K.14]

~~K.14~~ ~~The student will display gathered data in object graphs, picture graphs, and tables, and will answer questions related to the data.~~[Moved to K.11b]

Patterns, Functions, and Algebra

~~Focus: Attributes and Patterning~~

K.12~~15~~ The student will sort and classify objects according to attributes.

K.13~~16~~ The student will identify, describe, ~~and~~ extend, create, and transfer repeating patterns.

Grade One

The first-grade standards place emphasis on counting, sorting, and comparing sets of up to 1~~0~~0 objects; recognizing and describing simple repeating and growing patterns; and tracing, describing, and sorting plane ~~geometric~~ figures. Students' understanding of number will be expanded through learning and applying the basic addition facts through ~~the nines table~~sums of 20 and ~~the~~ corresponding subtraction facts; using nonstandard units to measure; and organizing and interpreting data. Fractional concepts will be expanded.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations.

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.~~

Number and Number Sense

~~Focus: Place Value and Fraction Concepts~~

1.1 The student will

a) count forward to 110~~from 0 to 100~~, starting at any number between 0 and 110, and write the corresponding numerals; ~~and~~

b) ~~group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value~~ [Moved to 1.2]represent a number from 0-110 using concrete objects;

c) count backward by ones when given any number between 1 and 30; [Moved from 1.2]and

d) count forward by ones, twos, fives, and tens to determine the total number of objects to 110. [Moved from 1.2]

1.2 The student, given up to 110 objects, ~~will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30.~~ [Moved to 1.1]

a) group a collection into tens and ones and write the corresponding numeral; [Moved from 1.1]

b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than*, *less than* or *equal to*; and

c) order three or fewer sets from least to greatest and greatest to least.

1.3 The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth, and the ordered position of each object. [Moved from K.3]

1.~~4~~3 The student will ~~identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.~~
a) investigate fractions by solving equal sharing problems with two, four, or eight sharers; and
b) verbally identify the parts of a region that represent fractions for halves, fourths, and eighths.

Computation and Estimation

~~Focus: Whole Number Operations~~

1.~~5~~4 The student, given a familiar problem situation involving magnitude, will
a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and
b) explain the reasonableness of the choice.

~~1.5~~ ~~The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.~~ [Moved to 1.7]

1.6 The student will create and solve ~~one~~single-step story and picture problems using ~~basic~~ addition facts with sums to ~~18~~ 20 or less and the corresponding subtraction facts.

1.~~7~~5 The student will ~~recall basic~~
a) recognize and describe with fluency part-whole relationships for numbers up to 10; and
b) demonstrate fluency with addition facts ~~with~~ for sums to 10~~18~~ or less and the corresponding subtraction facts.

Measurement and Geometry

~~Focus: Time and Nonstandard Measurement~~

1.~~8~~7 The student will
a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and
b) determine the value of a collection of like coins (pennies, nickels, or~~and~~ dimes) whose total value is 100 cents or less.

1.~~9~~8 The student will investigate the passage of time by
a) telling time to the hour and half-hour, using analog and digital clocks;~~.~~ and [Time to hour moved from K.9]
b) reading and interpreting a calendar. [Moved from 1.11]

1.~~10~~9 The student will use nonstandard units to measure and compare length, weight/mass, and volume. [Compare moved from 1.10]

~~1.10~~ ~~The student will compare, using the concepts of more, less, and equivalent,~~ [Compare moved to 1.10]
~~a) the volumes of two given containers; and~~ [Moved to 1.10 EKS]
~~b) the weight/mass of two objects, using a balance scale.~~ [Moved to 1.10 EKS]

~~1.11~~ ~~The student will use calendar language appropriately (e.g., names of the months, *today*, *yesterday*, *next week*, *last week*).~~ [Moved to K.9 and 1.9]

~~Geometry~~

~~Focus: Characteristics of Plane Figures~~

1.11~~12~~ The student will

a) identify and trace, describe, and sort plane ~~geometric~~ figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and ~~right~~ angles; and~~.~~

b) identify and describe representations of circles, squares, rectangles, and triangles in different environments and explain reasoning. [Moved from 1.13]

~~1.13~~ ~~The student will construct, model, and describe objects in the environment as geometric shapes (triangle, rectangle, square, and circle) and explain the reasonableness of each choice.~~ [Moved to 1.11b]

Probability and Statistics

~~Focus: Data Collection and Interpretation~~

1.12~~14~~ The student will

a) ~~investigate, identify, and describe~~collect, organize, and represent various forms of data collection ~~(e.g., recording daily temperature, lunch count, attendance, favorite ice cream),~~ using tables, picture graphs, and object graphs; and [Examples moved to US]

b) read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*. [Moved from 1.15]

~~1.15~~ ~~The student will interpret information displayed in a picture or object graph, using the vocabulary *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.~~ [Moved to 1.12b]

Patterns, Functions, and Algebra

~~Focus: Patterning and Equivalence~~

1.13~~16~~ The student will sort and classify concrete objects according to one or more attributes.~~,~~ ~~including color, size, shape, and thickness.~~ [Included in EKS]

1.14~~17~~ The student will identify~~recognize~~, describe, extend, ~~and~~ create, and transfer ~~a wide variety of~~ growing and repeating patterns. [Transfer included to match EKS bullet]

1.15~~18~~ The student will demonstrate an understanding of equality through the use of the equal sign.

Grade Two

The second-grade standards extend the study of number and spatial sense to include three-digit whole numbers and solid geometric figures. Students will continue to learn, use, and gain proficiency in ~~the basic~~ addition facts through ~~the tens table~~sums of 20 and ~~the~~ corresponding subtraction facts. Students will begin to use U.S. Customary and metric units of measure; predict, using simple probability; and create and interpret picture and bar graphs. Students will work with a variety of patterns and will develop ~~knowledge~~an understanding of equality ~~by identifying missing numbers in addition and subtraction facts~~.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations.

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.~~

Number and Number Sense

~~Focus: Place Value, Number Patterns, and Fraction Concepts~~

2.1 The student will

a) read, write, and identify the place value of each digit in a three-digit numeral, ~~using numeration~~ with and without models;

b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;

c) round two-digit numbers to the nearest ten; and [Reordered]

d) compare and order ~~two~~ whole numbers between 0 and 999~~, using symbols (>, <, or =) and words (*greater than*, *less than*, or *equal to*)~~. [Symbols and words included in EKS]

2.2~~4~~ The student will

a) count forward by twos, fives, and tens to 120~~100~~, starting at various multiples of 2, 5, or 10;

b) count backward by tens from 120~~100~~; and [Increased to 120 to correlate to Grade 1 change]

c) ~~recognize~~ use objects to determine whether a number is even or~~and~~ odd ~~numbers~~.

2.3~~2~~ The student will

a) count and identify the ordinal positions first through twentieth, using an ordered set of objects; and

b) write the ordinal numbers~~.~~ 1st through 20th. [Edited to match EKS].

2.~~4~~3 The student will
- a) name and write fractions represented by a set, region, or length model ~~identify the parts of a set and/or region that represent fractions~~ for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~;
- b) ~~write the~~ represent fractional~~s~~ parts with models and with symbols; and
- c) compare the unit fractions for halves, fourths, eighths, thirds, and~~fourths,~~ sixths~~, eighths, and tenths~~.

Computation and Estimation

~~Focus: Number Relationships and Operations~~

2.5 The student will ~~recall~~
- a) demonstrate fluency with addition facts ~~with~~ for sums to 20 or less and the corresponding subtraction facts; and
- b) recognize and use the relationships between addition and subtraction to solve single-step practical problems. [Moved from 2.9]

2.6 The student~~, given two whole numbers whose sum is 99 or less,~~ will
- a) estimate ~~the~~ sums and differences; ~~and~~ [Differences moved from 2.7a]
- b) ~~find the~~ determine sums and differences, using various methods ~~of calculation.~~; and [Differences moved from 2.7b]
- c) create and solve single-step and two-step practical problems involving addition and subtraction. [Moved from 2.8 and 2.21]

~~2.7 The student, given two whole numbers, each of which is 99 or less, will~~
- ~~a) estimate the difference; and~~ [Moved to 2.6a]
- ~~b) find the difference, using various methods of calculation.~~ [Moved to 2.6b]

~~2.8 The student will create and solve one- and two-step addition and subtraction problems, using data from simple tables, picture graphs, and bar graphs.~~ [Combined with 2.6c]

~~2.9 The student will recognize and describe the related facts that represent and describe the inverse relationship between addition and subtraction.~~ [Moved to 2.5b]

Measurement and Geometry

~~Focus: Money, Linear Measurement, Weight/Mass, and Volume~~

2.7~~10~~ The student will
- a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and
- b) ~~correctly~~ use the cent symbol ~~(¢)~~, dollar symbol ~~($)~~, and decimal point ~~(.)~~.

2.8~~11~~ The student will estimate and measure
- a) length to the nearest ~~centimeter and~~ inch; [Centimeters included in 3.8]
- b) weight/mass of objects ~~in~~ to the nearest pound~~s/ounces and kilograms/grams, using a scale; and~~. [Ounces, kilograms/grams included in 4.8]
- ~~c) liquid volume in cups, pints, quarts, gallons, and liters.~~ [Included in 3.8]

2.9~~12~~ The student will tell and write time to the nearest five minutes, using analog and digital clocks.

2.10~~13~~ The student will
- a) determine past and future days of the week; and
- b) identify specific days and dates on a given calendar.

2.11~~14~~ The student will read the temperature ~~on a Celsius and/or Fahrenheit thermometer~~ to the nearest 10 degrees. [Celsius/Fahrenheit included in EKS]

~~Geometry~~

~~Focus: Symmetry and Plane and Solid Figures~~

2.12~~15~~ The student will
a) draw a line of symmetry in a figure; and
b) identify and create figures with at least one line of symmetry.

2.13~~16~~ The student will identify, describe, compare, and contrast plane and solid ~~geometric~~ figures (circle/sphere, square/cube, and rectangle/rectangular prism).

Probability and Statistics

~~Focus: Applications of Data~~

2.14~~17~~ The student will ~~use data from experiments to construct picture graphs, pictographs, and bar graphs.~~
a) collect, organize, and represent data in picture graphs and bar graphs; and
b) read and interpret the data represented in picture graphs and bar graphs.

2.15~~18~~ The student will use data from experiments to predict outcomes when the experiment is repeated.

~~2.19~~ ~~The student will analyze data displayed in picture graphs, pictographs, and bar graphs.~~[Moved to 2.14b]

Patterns, Functions, and Algebra

~~Focus: Patterning and Numerical Sentences~~

2.16~~20~~ The student will identify, describe, create, ~~and~~ extend and transfer ~~a wide variety of~~ patterns found in objects, pictures, and numbers.

~~2.21~~ ~~The student will solve problems by completing numerical sentences involving the basic facts for addition and subtraction. The student will create story problems, using the numerical sentences.~~ [Moved to 2.5 and 2.6]

2.17~~22~~ The student will demonstrate an understanding of equality ~~by recognizing that~~ through the use of the equal symbol = ~~in an equation indicates equivalent quantities~~ and the use of the not equal symbol ≠ ~~indicates that quantities are not equivalent~~.

Grade Three

The third-grade standards place emphasis on ~~learning~~developing an understanding of, and solving problems that involve, multiplication and division ~~facts~~ through ~~the twelves table~~products of 100. ~~Students will be fluent in the basic addition facts through the tens table and the corresponding subtraction facts.~~ Students will apply the properties of addition and multiplication as strategies for solving problems. Concrete materials and two-dimensional representations will be used to introduce addition and subtraction with fractions and the concept of probability as the measurement of chance. Students will use standard units (U.S. Customary and metric) to measure temperature, length, and liquid volume~~, and weight~~ and identify relevant properties of shapes, points, line segments, rays, angles, vertices, and lines. Students will identify polygons with 10 or fewer sides, combine and subdivide polygons, and name the resulting polygons. ~~investigate and describe the identity and commutative properties for addition and multiplication.~~

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations.

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.~~

Number and Number Sense

~~Focus: Place Value and Fractions~~

3.1 The student will

a) read and write six-digit numerals and identify the place ~~value~~ and value of each digit;
b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and
c) compare ~~two~~ and order whole numbers ~~between 0 and~~each 9,999 or less~~, using symbols (>, <, or =) and words (*greater than*, *less than*, or *equal to*)~~. [Symbols and words included in EKS]

~~3.2~~ ~~The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. The student will use these relationships to solve problems.~~ [Addition/subtraction included in 2.5; multiplication/division moved to 3.4 EKS]

3.2~~3~~ The student will

a) name and write fractions (including improper fractions and mixed numbers) represented by a model;
b) ~~model~~ represent fractions (including improper fractions and mixed numbers) ~~and write the fractions' names~~with models and symbols; and
c) use models to compare proper fractions having like and unlike denominators, using words and symbols (>, <, ~~or~~ =, or ≠).

Computation and Estimation

~~Focus: Computation and Fraction Operations~~

3.3~~4~~ The student will

a) estimate ~~solutions to~~ and determine the sum or difference of two whole numbers; and

b) create and solve single-step and multistep practical problems involving ~~the~~ sums or differences of two whole numbers, each 9,999 or less~~, with or without regrouping~~.

3.4~~5~~ The student will ~~recall multiplication facts through the twelves table, and the corresponding division facts.~~[Recall moved to 4.4a]

a) represent multiplication and division, using a variety of approaches and models; [Moved from 3.6]

b) create and solve single-step practical problems that involve multiplication and division through 10 x 10; and [Create and solve multiplication moved from 3.6; create and solve with division is new expectation]

c) create and solve single-step practical problems that involve multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less. [Moved from 3.6]

~~3.6~~ ~~The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.~~[Moved to 3.4a and 3.4c]

3.5~~7~~ The student will solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less.

Measurement and Geometry

~~Focus: U.S. Customary and Metric Units, Area and Perimeter, and Time~~

3.6~~8~~ The student will

a) determine~~, by counting,~~ the value of a collection of bills and coins whose total value is $5.00 or less~~,~~;

b) compare the value of ~~the~~ two sets of coins or two sets of coins and bills ~~and coins,~~; and

c) make change from $5.00 or less. [Edited to match EKS]

3.7~~9~~ The student will estimate and use U.S. Customary and metric units to measure

a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter; and

b) liquid volume in cups, pints, quarts, gallons, and liters~~;~~.

~~c) weight/mass in ounces, pounds, grams, and kilograms; and~~ [Included in 4.8]

~~d) area and perimeter.~~ [Moved to 3.8a,b]

3.8~~10~~ The student will estimate and [Moved from 3.9d EKS]

a) measure the distance around a polygon in order to determine its perimeter using U.S. Customary and metric units[Moved from 3.9d EKS]; and

b) count the number of square units needed to cover a given surface in order to determine its area.

3.9~~11~~ The student will

a) tell time to the nearest minute, using analog and digital clocks; and

b) ~~determine~~ solve practical problems related to elapsed time in one-hour increments ~~over~~ within a 12-hour period; and

c) identify equivalent periods of time and solve practical problems related to equivalent periods of time. [Moved from 3.12]

~~3.12~~ ~~The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.~~[Moved to 3.9c]

3.10~~13~~ The student will read temperature to the nearest degree ~~from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used~~. [Thermometer information moved to EKS]

3.11 The student will
a) define polygon; [Moved from 4.12a]
b) identify and name polygons with 10 or fewer sides; and~~.~~[Moved from 4.12b]
c) combine and subdivide polygons with 3 or 4 sides and name the resulting polygons.

~~3.14~~ ~~The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.~~ [Moved to 4.11]

3.12~~15~~ The student will identify and draw representations of points, lines, line segments, rays, and angles~~, and lines~~.

3.13~~16~~ The student will identify and describe congruent and noncongruent ~~plane~~ figures.

Probability and Statistics

~~Focus: Applications of Data and Chance~~

3.14~~17~~ The student will
a) collect~~, and~~ organize, and represent data~~, using observations, measurements, surveys, or experiments~~[Included in EKS] in picture graphs or bar graphs; and~~;~~
b) ~~construct a line plot, a picture graph, or a bar graph to represent the data; and~~[Line plot moved to 5.16; construct picture and bar graphs moved to 3.14a]
~~c)~~ read and interpret the data represented in bar graphs~~,~~ and picture graphs ~~and write a sentence analyzing the data~~.

3.15~~18~~ The student will investigate and describe the concept of probability as a measurement of chance and list possible ~~results~~ outcomes ~~of~~for a given situation.

Patterns, Functions, and Algebra

~~Focus: Patterns and Property Concepts~~

3.16~~19~~ The student will ~~recognize and~~ identify, describe, create, extend and transfer patterns found in objects, pictures, numbers and tables. ~~a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.~~

3.17~~20~~ The student will create equations to represent equivalent mathematical relationships. [Moved from 3.20 EKS]
~~a) investigate the identity and the commutative properties for addition and multiplication; and~~ [Use of properties moved to 3.3 EKS and 3.4 EKS]
~~b) identify examples of the identity and commutative properties for addition and multiplication.~~

Grade Four

The fourth-grade standards place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of fractions and decimals~~ by finding common multiples and factors~~. Students will be fluent in the basic multiplication facts through the twelves table and the corresponding division facts as they become proficient in multiplying larger numbers. Students will apply the properties of addition and multiplication as strategies for solving problems. Students also will refine their estimation skills for computations and measurements. Students will identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. Concrete materials and two-dimensional representations will be used to solve problems involving perimeter, patterns, probability, and equivalence of fractions and decimals. Students will recognize images of figures resulting from geometric transformations, such as reflection, translation, and rotation. ~~Students will investigate and describe the associative property for addition and multiplication.~~

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations.

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.~~

Number and Number Sense

~~Focus: Place Value, Fractions, and Decimals~~

4.1 The student will

a) identify orally and in writing the place and value for each digit in a whole number expressed through millions;
b) compare and order ~~two~~ whole numbers expressed through millions~~, using symbols (>, <, or =)~~; and [Symbols included in EKS]
c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

4.2 The student will

a) compare and order fractions and mixed numbers, with and without models;
b) represent equivalent fractions; and
c) identify the division statement that represents a fraction with models and in context.

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number~~, tenth, and hundredth~~; [Round to tenth and hundredth included in 5.1]
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.

Computation and Estimation

~~Focus: Factors and Multiples, and Fraction and Decimal Operations~~

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 x 12, and the corresponding division facts; [Moved from 3.5]~~estimate sums, differences, products, and quotients of whole numbers;~~

b) estimate and determine sums, differences, and products of ~~add, subtract, and multiply~~ whole numbers;

c) estimate and determine quotients of ~~divide~~ whole numbers, ~~finding quotients~~ with and without remainders; and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division ~~problems~~ with whole numbers.

4.5 The student will

a) determine common multiples and factors, including least common multiple and greatest common factor;

b) add and subtract fractions and mixed numbers having like and unlike denominators ~~that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors~~; and

~~c) add and subtract with decimals; and~~ [Moved to 4.6a]

c~~d~~) solve single-step and multistep practical problems involving addition and subtraction with fractions ~~and with decimals~~. [Decimals limited to single-step only and moved to 4.6b; solve multistep with decimals included in 5.5b]

4.6 The student will

a) add and subtract with decimals; and [Moved from 4.5c]

b) solve single-step practical problems involving addition and subtraction with decimals. [Moved from 4.5d; decimals changed to single-step only; multistep with decimals included in 5.5b]

Measurement and Geometry

~~Focus: Equivalence within U.S. Customary and Metric Systems~~

~~4.6 The student will~~

~~a) estimate and measure weight/mass and describe the results in U.S. Customary and metric units as appropriate; and~~[Moved to 4.8b]

~~b) identify equivalent measurements between units within the U.S. Customary system~~[Moved to 4.8c]~~(ounces, pounds, and tons)~~[Moved to 4.8 EKS] ~~and between units within the metric system (grams and kilograms).~~ [Included in 5.9a]

~~4.7 The student will~~

~~a) estimate and measure length, and describe the result in both metric and U.S. Customary units; and~~ [Moved to 4.8a]

~~b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles)~~[Moved to 4.8c] ~~and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters).~~[Included in 5.9a]

4.7 The student will solve practical problems that involve determining perimeter and area in standard units of measure.

4.8 The student will
a) estimate and measure length and describe the result in U.S. Customary and metric units; and[Moved from 4.7a]
~~a~~b) ~~estimate and measure liquid volume and describe the results in U.S. Customary units~~[Included in 3.7b]estimate and measure weight/mass and describe the results in U.S. Customary and metric units[Moved from 4.6a]; ~~and~~
~~b~~c) identify equivalent measure~~ments~~ of length, weight/mass and liquid volume between units within the U.S. Customary system ~~(cups, pints, quarts, and gallons)~~[Moved to EKS]~~.~~; and
d) solve practical problems that involve measuring length, weight/mass, and liquid volume in U.S. Customary.

4.9 The student will ~~determine~~ solve practical problems related to elapsed time in hours and minutes within a 12-hour period.

4.10 The student will
a) identify and describe ~~representations of~~ points, lines, line segments, rays, and angles, including endpoints and vertices; and
b) identify and describe ~~representations of lines that illustrate~~ intersecting~~on~~, parallel~~ism~~, and perpendicular~~ity~~ lines.

~~4.11 The student will~~
~~a) investigate congruence of plane figures after geometric transformations, such as reflection, translation, and rotation, using mirrors, paper folding, and tracing; and~~
~~b) recognize the images of figures resulting from geometric transformations, such as translation, reflection, and rotation.~~ [Moved to 5.14]

4.11 The student will identify, describe, compare, and contrast characteristics of plane and solid figures by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models. [Moved from 3.14]

4.12 The student will classify quadrilaterals as a parallelograms, rectangles, squares, rhombuses, and/or trapezoids.
~~a) define polygon; and~~
~~b) identify polygons with 10 or fewer sides.~~[Polygons other than quadrilaterals moved to 3.11]

~~Geometry~~
~~Focus: Representations and Polygons~~

Probability and Statistics
~~Focus: Outcomes and Data~~

4.13 The student will
a) ~~predict~~ determine the likelihood of an outcome of a simple event; and
b) represent probability as a number between 0 and 1, inclusive; and
c) create a model or practical problem to represent a given probability.

4.14 The student will
a) collect, organize, and represent data in a bar graph and line graph~~display.~~; and
b) make observations and inferences about data represented in bar graphs and line graphs.~~interpret data from a variety of graphs.~~

Patterns, Functions, and Algebra

~~Focus: Geometric Patterns, Equality, and Properties~~

4.15 The student will ~~recognize~~identify, describe, create, and extend ~~numerical and geometric~~ patterns found in objects, pictures, numbers, and tables.

4.16 The student will

~~a)~~ recognize and demonstrate the meaning of equality in an equation.~~; and~~

~~b) investigate and describe the associative property for addition and multiplication.~~ [Moved to 5.19]

Grade Five

The fifth-grade standards place emphasis on number sense with whole numbers, fractions, and decimals. This focus includes concepts of prime and composite numbers, identifying even and odd numbers, and solving problems using order of operations for positive whole numbers. Students will develop proficiency in the use of fractions and decimals to solve problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space or tree diagram. Students also will solve problems involving volume, area, and perimeter. Students will be introduced to variable expressions ~~and open sentences, and will model one-step linear equations in one variable, using addition and subtraction~~. Students will investigate and ~~recognize~~identify the properties of addition and multiplication ~~the distributive property~~. All of these skills assist in the development of the algebraic concepts needed for success in the middle grades.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations.

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.~~

Number and Number Sense

~~Focus: Prime and Composite Numbers and Rounding Decimals~~

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

5.2 The student will

a) ~~recognize and name~~ represent and identify equivalencies among fractions and ~~in their equivalent~~ decimals, with and without models; ~~form and vice versa;~~ and
b) compare and order fractions and decimals in a given set from least to greatest and greatest to least.

5.3 The student will

a) identify and describe the characteristics of prime and composite numbers; and
b) identify and describe the characteristics of even and odd numbers.

Computation and Estimation

~~Focus: Multistep Applications and Order of Operations~~

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers.

5.5 The student will
- a) ~~find the sum, difference,~~determine the product~~,~~ and quotient of two numbers ~~expressed as~~involving decimals ~~decimals through thousandths (divisors with only one nonzero digit)~~; and
- b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and single-step practical problems involving division of decimals.

5.6 The student will
- a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and ~~express answers in simplest form.~~ [Included in EKS]
- b) solve single-step practical problems that involve modeling and determining the product of a whole number and a unit fraction.

5.7 The student will evaluate whole number numerical expressions, using the order of operations limited to parentheses, addition, subtraction, multiplication, and division.

Measurement and Geometry

~~Focus: Perimeter, Area, Volume, and Equivalent Measures~~

5.8 The student will
- a) ~~find~~determine perimeter, area, and volume in standard units of measure; and
- b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation~~;~~.
- ~~c) identify equivalent measurements within the metric system;~~ [Moved to 5.9a]
- ~~d) estimate and then measure to solve problems, using U.S. Customary and metric units; and~~ [U.S. Customary included in 4.8d; Metric moved to 5.9b]
- ~~e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.~~ [Moved to 5.9 EKS]

5.9 The student will
- a) identify equivalent measurements within the metric system; and [Moved from 5.8c]
- b) solve practical problems involving length, mass, and liquid volume using metric units. [Moved from 5.8d]

5.10~~9~~ The student will identify and describe the diameter, radius, chord, and circumference of a circle.

5. 11~~10~~ The student will ~~determine an amount of~~ solve practical problems related to elapsed time in hours and minutes within a 24-hour period.

5. 12~~11~~ The student will classify and measure right, acute, obtuse, and straight angles. [Classify moved from 5.12a]

~~Geometry~~

~~Focus: Classification and Subdividing~~

5. 13~~12~~ The student will ~~classify~~
- ~~a) angles as right, acute, obtuse, or straight; and~~ [Classification of angles moved to 5.12]
- ~~b~~a) classify triangles as right, acute, or obtuse ,and equilateral, scalene, or isosceles; and
- b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

5.14~~13~~ The student~~, using plane figures (square, rectangle, triangle, parallelogram, rhombus, and trapezoid),~~ will
- a) ~~develop definitions of these plane figures~~[Included in 4.12]recognize and apply transformations, such as translation, reflection, and rotation;[Moved from 4.11b] and
- b) investigate and describe the results of combining and subdividing ~~plane figures~~polygons.

Probability and Statistics

~~Focus: Outcomes and Measures of Center~~

5.15~~14~~ The student will ~~make predictions and~~ determine the probability of an outcome by constructing a sample space.

5. 16~~15~~ The student, given a practical problem~~ situation,~~ will ~~collect, organize, and interpret data in a variety of forms, using~~
- a) represent data in line plots, ~~a~~ stem-and-leaf plots, and line graphs; and [Line plot moved from 3.17]
- b) make observations and inferences about data represented in a line plot, stem-and-leaf plot, and line graph. [Line plot moved from 3.17]

5.17~~16~~ The student will solve practical problems that involve
- a) describ~~e~~ing mean, median, and mode as measures of center;
- b) describ~~e~~ing mean as fair share;
- c) describing the range of a set of data as a measure of spread; and [Reordered]
- d~~c~~) ~~find~~ determine the mean, median, mode, and range of a set of data. ~~; and~~
- ~~d) describe the range of a set of data as a measure of variation.~~ [Reordered and edited]

Patterns, Functions, and Algebra

~~Focus: Equations and Properties~~

5.18~~17~~ The student will describe and express the relationship of ~~found in a~~ number patterns ~~and express the relationship~~found in objects, pictures, numbers and tables.

5.19~~18~~ The student will
- a) investigate and describe the concept of variable;
- b) write an ~~open sentence~~equation to represent a given mathematical relationship, using a variable;
- c) use a variable expression to represent a verbal quantitative expression involving one operation; and ~~model one-step linear equations in one variable, using addition and subtraction; and~~ [Moved to 6.14]
- d) create a problem situation based on a given ~~open sentence~~equation, using a single variable.

5.20~~19~~ The student will investigate and identify~~recognize the distributive property of multiplication over addition.~~
- a) the identity properties of addition and multiplication; [Moved from 3.20]
- b) the commutative properties of addition and multiplication; [Moved from 3.20] and
- c) the associative properties of addition and multiplication; and [Moved from 4.16b]
- d) the distributive property of multiplication over addition. [Reordered]

Grade Six

The sixth-grade standards ~~are~~ provide a transition from the emphasis placed on whole number arithmetic in the elementary grades to foundations of algebra. The standards ~~emphasize~~ include a focus on rational numbers and operations involving rational numbers. Students will use ratios to compare data sets; recognize decimals, fractions, and percents as ratios; solve single-step and multistep problems, using positive rational numbers; and gain a foundation in the understanding of and operations with integers. Students will solve problems involving area and perimeter, and begin to graph in a coordinate plane. In addition, students will build on the concept of graphical representation of data developed in the elementary grades and develop concepts regarding measures of center. Students will solve linear equations and inequalities in one variable, and use algebraic terminology. Students will represent proportional relationships using two variables as a precursor to the development of the concept of linear functions. ~~Students will solve problems involving area, perimeter, and surface area, work with π (pi), and focus on the relationships among the properties of quadrilaterals. In addition, students will focus on applications of probability and statistics.~~

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations. ~~Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.~~

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.~~

Number and Number Sense

~~Focus: Relationships among Fractions, Decimals, and Percents~~

6.1 The student will describe and compare ~~data~~sets, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

6.2 The student will

a) ~~investigate and describe fractions, decimals, and percents as ratios~~ [Moved to EKS] represent and determine equivalencies among fractions, decimals, and percents; and

~~b)~~ ~~identify a given fraction, decimal, or percent from a representation;~~ [Moved to EKS]

~~c)~~ ~~demonstrate equivalent relationships among fractions, decimals, and percents; and~~

~~d~~b) compare and order ~~fractions, decimals, and percents~~positive rational numbers.

6.3 The student will

a) identify and represent integers;

b) order and compare integers; and

c) identify and describe absolute value of integers.

~~6.4~~ ~~The student will demonstrate multiple representations of multiplication and division of fractions.~~[Moved to 6.5 EKS]

6.4~~5~~ The student will ~~investigate and describe concepts of positive~~recognize and represent patterns with whole number exponents and perfect squares.

Computation and Estimation

~~Focus: Applications of Operations with Rational Numbers~~

6.5~~6~~ The student will

a) multiply and divide fractions and mixed numbers; ~~and~~

b) ~~estimate solutions and then~~ solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers;~~.~~ and

c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]

~~6.7~~ ~~The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals.~~ [Moved to 6.5c]

6.6 The student will

a) add, subtract, multiply, and divide integers; and [Moved from 7.3]

b) solve practical problems involving operations with integers. [Moved from 7.3]

6.7~~8~~ The student will ~~evaluate whole numbers~~simplify numerical expressions involving integers, using the order of operations.

Measurement and Geometry

~~Focus: Problem Solving with Area, Perimeter, Volume, and Surface Area~~

~~6.9~~ ~~The student will make ballpark comparisons between measurements in the U.S. Customary System of measurement and measurements in the metric system.~~ [Included in 7.3 EKS]

6.8~~10~~ The student will

a) ~~define~~ derive π (pi) ~~as the ratio of the circumference of a circle to its diameter~~;

b) solve problems, including practical problems, involving circumference and area of a circle~~, given the diameter or radius~~; and

c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles~~; and~~.

~~d) describe and determine the volume and surface area of a rectangular prism.~~[Included in 7.4a]

~~Geometry~~

~~Focus: Properties and Relationships~~

6.9~~11~~ The student will

a) identify the ~~coordinates of a point in a~~components of the coordinate plane; and

b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

6.10~~12~~ The student will determine congruence of segments, angles, and polygons.

~~6.13~~ ~~The student will describe and identify properties of quadrilaterals.~~[Moved to 7.6]

Probability and Statistics

~~Focus: Practical Applications of Statistics~~

6.11~~14~~ The student, given a ~~problem~~ practical situation, will

a) ~~construct~~ represent data in a circle graph~~s~~;

b) ~~draw conclusions and~~ make ~~predictions,~~observations and inferences about data represented in a ~~using~~ circle graph~~s~~; and

c) compare ~~and contrast~~ circle graphs with the same data represented in bar graphs, picture graphs, and line plots ~~that present information from the same data set~~.

6.12~~15~~ The student will

a) ~~describe~~ represent the mean of a data set graphically as the balance point; ~~and~~

b) ~~decide which~~determine an appropriate measure of center ~~is appropriate~~ for a given ~~purpose~~situation; and

c) determine the impact on measures of center when a single value of a data set is added, removed, or changed. [Moved from 5.16]

~~6.16 The student will~~

~~a) compare and contrast dependent and independent events; and~~ [Moved to 8.11a]

~~b) determine probabilities for dependent and independent events.~~ [Included in 8.11b]

Patterns, Functions, and Algebra

~~Focus: Variable Equations and Properties~~

~~6.17 The student will identify and extend geometric~~ [Included in AFDA.1 and AII.5] ~~and arithmetic~~ [Included in AI.7] ~~sequences.~~

6.13 The student will

a) write an equation in the form $y = kx$, where k is the constant of proportionality, to represent a proportional relationship between two quantities, including practical problems; and

b) make connections among representations of a proportional relationship between two quantities using verbal descriptions, tables, graphs, and equations.

6.14~~18~~ The student will solve one-step linear equations in one variable, ~~involving whole number coefficients and positive rational solutions~~including practical problems that require the solution of a one-step linear equation in one variable.

~~6.19 The student will investigate and recognize~~

~~a) the identity properties for addition and multiplication;~~

~~b) the multiplicative property of zero; and~~

~~c) the inverse property for multiplication.~~

[Included in the EKS for 6.7, 6.14, 6.15]

6.15~~20~~ The student will

a) represent a practical situation with a ~~graph inequalities~~ linear inequality in one variable; and

b) solve one-step linear inequalities in one variable and graph the solution on a number line.

Grade Seven

The seventh-grade standards continue to emphasize the foundations of algebra. ~~Students who successfully complete the seventh-grade standards should be prepared to study Algebra I in grade eight.~~ The standards address the concept of and operations with rational numbers by continuing their study from sixth-grade. Students will build on the concept of ratios to solve problems involving ~~Topics in grade seven include~~ proportional reasoning~~, . integer computation,~~Students will solve problems involving volume and surface area and focus on the relationships among the properties of quadrilaterals. Probability is investigated through comparing experimental results to theoretical expectations. Students continue to develop their understanding of ~~–~~solving ~~two-step~~ linear equations and inequalities in one variable~~,~~ by applying the properties of real numbers. ~~and recognizing different representations for~~ Students discern between proportional and non-proportional relationships and begin to develop a concept of slope as rate of change. ~~Students will apply the properties of real numbers in solving equations, solve inequalities, and use data analysis techniques to make inferences, conjectures, and predictions.~~

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations. ~~Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.~~

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types~~

Number and Number Sense

~~Focus: Proportional Reasoning~~

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;
c) compare and order ~~fractions, decimals, percents and numbers written in scientific notation~~rational numbers;
d) determine square roots of perfect squares; and
e) identify and describe absolute value ~~for~~ of rational numbers.

~~7.2~~ ~~The student will describe and represent arithmetic~~ [Included in AI.7]~~and geometric sequences~~ [Included in AFDA.1 and AII.5]~~using variable expressions.~~

Computation and Estimation

~~Focus: Integer Operations and Proportional Reasoning~~

7.2~~3~~ The student will

a) ~~model addition, subtraction, multiplication and division of integers~~ [Moved to 6.6a] simplify numerical expressions involving rational numbers using the order of operations; [Moved from 8.1a] and

b) ~~add, subtract, multiply, and divide integers~~ [Moved to 6.6b] solve practical problems involving operations with rational numbers.

7.3~~4~~ The student will solve single-step and multistep practical problems, using proportions~~al reasoning~~.

Measurement and Geometry

~~Focus: Proportional Reasoning~~

7.4~~5~~ The student will

a) describe and determine the volume and surface area of rectangular prisms and cylinders; and

b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.~~; and~~

~~c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.~~[Included in 8.6b]

7.5~~6~~ The student will ~~determine whether plane figures—quadrilaterals and triangles—are similar and write proportions to express the relationships between~~solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar ~~figures~~quadrilaterals and triangles.

~~Geometry~~

~~Focus: Relationships between Figures~~

7.6~~7~~ The student will

a) compare and contrast ~~the following~~ quadrilaterals based on their properties~~:~~; and ~~parallelogram, rectangle, square, rhombus, and trapezoid.~~

b) solve problems to determine unknown side lengths or angle measures of quadrilaterals.

7.7~~8~~ The student~~, given a polygon in the coordinate plane,~~ will apply ~~represent transformations (~~translations and reflections of right triangles or rectangles~~, dilations, rotations, and translations)by graphing~~ in the coordinate plane.

Probability and Statistics

~~Focus: Applications of Statistics and Probability~~

7.8~~9~~ The student will

a) determine the theoretical and experimental probabilities of an event; and

b) investigate and describe the difference between the experimental probability and theoretical probability of an event.

~~7.10 The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle.~~ [Moved to 5.15]

7.9~~11~~ The student, given data for a practical situation, will
a) ~~construct and analyze~~represent data in a histograms~~;~~ ~~and~~
b) make observations and inferences about data represented in a histogram; and
c) compare ~~and contrast~~ histograms with ~~other types of graphs presenting information from~~ the same data ~~set~~represented in stem-and-leaf plots, line plots, and circle graphs.

Patterns, Functions, and Algebra

~~Focus: Linear Equations~~

~~7.12~~ ~~The student will represent relationships with tables, graphs, rules, and words.~~[Included 7.10]

7.10 The student will
a) determine if a proportional relationship exists between two quantities, given a table of values or a graph;
b) identify the unit rate of a proportional relationship between two quantities represented by a table, graph, equation, or verbal description, including those in practical situations.
c) determine the slope as rate of change in a proportional relationship, given two ordered pairs; and
d) graph a line representing a proportional relationship given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change.

7.11~~13~~ The student will
~~a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and~~[Included in 7.12 EKS]
b~~)~~ evaluate algebraic expressions for given replacement values of the variables.

7.12~~14~~ The student will
~~a)~~ solve ~~one- and-~~ [Included in 6.14]two-step linear equations in one variable~~; and~~, including
~~b) solve~~ practical problems that require~~ing~~ the solution of a ~~one- and-~~ two-step linear equation~~s~~ in one variable.

7.13~~15~~ The student will
~~a)~~ solve one- and two-step [two-step moved from 8.15b]linear inequalities in one variable, including practical problems~~;~~, and
~~b)~~ graph the solution~~s to inequalities~~ on ~~the~~ a number line.

~~7.16~~ ~~The student will apply the following properties of operations with real numbers:~~
~~a) the commutative and associative properties for addition and multiplication;~~
~~b) the distributive property;~~
~~c) the additive and multiplicative identity properties;~~
~~d) the additive and multiplicative inverse properties; and~~
~~e) the multiplicative property of zero.~~
[Incorporated into 7.2, 7.12, and 7.13]

Grade Eight

The eighth-grade standards continue to build on the concepts needed for success in high school level algebra and geometry~~are intended to serve two purposes. First, the standards contain content that reviews or extends concepts and skills learned in previous grades. Second, they contain new content that prepares students for more abstract concepts in algebra and geometry. The eighth-grade standards provide students additional instruction and time to acquire the concepts and skills necessary for success in Algebra I.~~ Students will ~~gain proficiency in computation with rational~~ explore real numbers and the subsets of the real number system. ~~and will use p~~Proportions~~s~~al reasoning is expounded upon as students ~~to~~ solve a variety of problems. Students find the volume and surface area of more complex three-dimensional figures and apply transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem creating a foundation for further study of triangular relationships in geometry. ~~New~~ Students represent data, both univariate and bivariate data, and make predictions by observing data patterns. Students build upon the algebraic concepts developed in courses that the standards for Grades 6 and 7 Mathematics, include simplifying algebraic expressions, solving multistep equations and inequalities, and graphing linear ~~equations~~functions~~, visualizing three-dimensional shapes represented in two-dimensional drawings, and applying transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem and represent relations and functions, using tables, graphs, and rules~~. The eighth-grade standards ~~provide a more solid~~are vital to providing a solid foundation in Algebra I for ~~those~~ students ~~not ready for Algebra I in grade eight~~in middle school mathematics.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. ~~While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers.~~ However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts ~~and relationships~~ or for proficiency in basic computations. ~~Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.~~

~~Mathematics has its own language, and t~~The acquisition of specialized mathematical vocabulary and language ~~patterns~~ is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving. ~~Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.~~

Problem solving ~~has been~~is integrated throughout the ~~six~~ content strands. The development of problem-solving skills ~~should be~~is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be ~~Instruction in the process of problem solving will need to be~~ integrated early and continuously into each student's mathematics education. ~~Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types~~

Number and Number Sense

~~Focus: Relationships within the Real Number System~~

8.1 The student will

- ~~a)~~ ~~simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and~~ [Combined with 7.2]
- ~~b)~~ compare and order real numbers~~decimals, fractions, percents, and numbers written in scientific notation.~~[Moved to EKS]

8.2 The student will describe ~~orally and in writing~~ the relationships between the subsets of the real number system.

8.3 The student will
- a) estimate and determine the two consecutive integers between which a square root lies; and [Moved from 8.5b]
- b) determine both the positive and negative square roots of a given perfect square. [Moved from 8.5 EKS]

Computation and Estimation

~~Focus: Practical Applications of Operations with Real Numbers~~

8. ~~4~~3 The student will
- a) solve practical problems involving rational numbers, percents, ratios, and proportions; and
- b) determine the percent increase or decrease for a given situation.

~~8.4~~ ~~The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables.~~ [Moved to 8.14a]

~~8.5~~ ~~The student will~~
- ~~a) determine whether a given number is a perfect square; and~~[Included in 7.1]
- ~~b) find the two consecutive whole numbers between which a square root lies.~~ [Included in 8.3a]

Measurement and Geometry

~~Focus: Problem Solving~~

8.5~~6~~ The student will use
- ~~a) verify by measuring and describe~~ the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles~~; and~~
- ~~b)~~ to determine the measure of unknown angles ~~of less than 360°~~.

8.6~~7~~ The student will
- a) ~~investigate and~~ solve problems, including practical problems, involving volume and surface area of rectangular prisms, cylinders, cones, and square-based pyramids; and
- b) describe how changing one measured attribute of a rectangular prism~~figure~~ affects the volume and surface area.

~~Geometry~~

~~Focus: Problem Solving with 2- and 3-Dimensional Figures~~

8.7~~8~~ The student will
- a) given a polygon, apply transformations, to include translations, rotations, reflections, and dilations, in the coordinate ~~to~~ plane ~~figures~~; and
- b) identify practical applications of transformations.

8.8~~9~~ The student will construct a three-dimensional model, given the top or bottom, side, and front views.

8.9~~10~~ The student will
- a) verify the Pythagorean Theorem; and
- b) apply the Pythagorean Theorem.

8.10~~11~~ The student will solve ~~practical~~ area and perimeter problems, including practical problems, involving composite plane figures.

Probability and Statistics

~~Focus: Statistical Analysis of Graphs and Problem Situations~~

8.11~~12~~ The student will

a) compare and contrast the probability of independent and dependent events; and [Moved from 6.16]~~with and without replacement.~~

b) determine probabilities~~y~~ for independent and dependent events.

8.12 The student will

a) represent data in boxplots; [Moved from A.10]

b) make observations and inferences about data represented in boxplots; and [Moved from A.10]

c) compare boxplots with the same data represented in histograms, stem and leaf plots, and line plots. [Moved from A.10]

8.13 The student will

a) represent data in scatterplots~~make comparisons, predictions, and inferences, using information displayed in graphs~~; and

b) make observations and inferences about data represented in ~~construct and analyze~~ scatterplots.

Patterns, Functions, and Algebra

~~Focus: Linear Relationships~~

8.14~~4~~ The student will

a) ~~apply the order of operations to~~ evaluate an algebraic expression~~s~~ for given replacement values of the variables[Moved from Computation and Estimation strand 8.4]; and

b) simplify expressions in one variable.

8.15~~14~~ The student will ~~make connections between any two representations (tables, graphs, words, and rules) of a given relationship.~~

a) determine whether a given relation is a function; and

b) determine the domain and range of a function. [Moved from 8.17]

8.17~~15~~ The student will

~~a)~~ solve multistep linear equations in one variable with the variable on one and both ~~two~~ sides of the equation, including practical problems requiring the solution of a multistep linear equation in one variable.~~;~~

~~b) solve two-step linear inequalities and graph the results on a number line; and~~ [Moved to 8.18]

~~c) identify properties of operations used to solve an equation.~~[Incorporated into 8.14, 8.17, and 8.18 EKS]

8.16 The student will

a) identify the slope and y-intercept of a linear function given the graph or given the equation in $y = mx + b$ form;

b) graph a linear function given the equation in $y = mx + b$ form; and~~two variables.~~

c) distinguish between proportional and non-proportional situations modeled by linear functions.

~~8.17 The student will identify the domain, range,~~ [Moved to 8.15b]~~independent variable, or dependent~~ [Moved to 8.16 EKS]~~variable in a given situation.~~

8.18 The student will solve multistep linear inequalities in one variable with the variable on one and both sides of the inequality symbol, including practical problems, and graph the results on a number line. [Moved from 8.15b]

Algebra I

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. ~~The standards below outline the content for a one-year course in Algebra I.~~ All students are expected to achieve the Algebra I standards. The study of Algebra I assists students in generalizing patterns or modeling relevant, practical situations with algebraic models. In order to assist students in developing meaning and connecting algebraic concepts to geometry and statistics, ~~When planning for instruction,~~ consideration ~~will~~ should be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the ~~arithmetic~~ numeric to the symbolic. ~~Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics.~~ Connections between Algebra I and ~~also should be made to~~ other subject areas through practical applications~~. This approach to teaching algebra should~~ may assist in helping students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze behaviors of functions. These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

~~Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.~~

~~Throughout the course, students should be encouraged to engage in discourse about mathematics with teachers and other students, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop confidence in themselves as mathematics students.~~

Expressions and Operations

A.1 The student will
- a) represent verbal quantitative situations algebraically; and
- b) evaluate ~~these~~ algebraic expressions for given replacement values of the variables.

A.2 The student will perform operations on polynomials, including
- a) applying the laws of exponents to perform operations on expressions;
- b) adding, subtracting, multiplying, and dividing polynomials; and
- c) factoring completely first- and second-degree binomials and trinomials in one ~~or two~~ variable~~s~~. ~~Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.~~[Moved to EKS]

A.3 The student will simplify~~express the~~
- a) square roots of non-negative rational numbers and monomial algebraic expressions;
- b) cube roots of ~~whole~~ rational numbers; and ~~the square root of a monomial algebraic expression in simplest radical form~~
- c) numerical expressions containing square or cube roots.

Equations and Inequalities

A.4 The student will solve ~~multistep linear and quadratic equations in two variables, including~~
- a) multistep linear equations in one variable algebraically[Reordered]~~solving literal equations (formulas) for a given variable~~ [Reordered];
- ~~b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets~~[Moved to EKS];
- ~~c~~b) ~~solving~~ quadratic equations in one variable algebraically ~~and graphically~~;
- ~~d~~c) ~~solving~~ literal equations for a specified variable[Reordered]~~multistep linear equations algebraically and graphically~~ [Reordered];
- ~~e~~d) ~~solving~~ systems of two linear equations in two variables algebraically and graphically; and
- ~~f~~e) ~~solving real-world~~practical problems involving equations and systems of equations.

~~Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.~~ [Included in EKS]

A.5 The student will ~~solve multistep linear inequalities in two variables, including~~
- a) solve~~ing~~ multistep linear inequalities in one variable algebraically and represent the solution graphically;
- b) represent the solution of linear inequalities in two variables algebraically and graphically [Moved from SOL stem]~~justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets~~ [Included in EKS];
- c) solve~~ing real-world~~ practical problems involving inequalities; and
- d) solve~~ing~~ systems of inequalities algebraically and graphically.

A.6 The student will ~~graph linear equations and linear inequalities in two variables, including~~ [Linear equations moved to A.6c and linear inequalities moved to A.5b]
- a) determine~~ing~~ the slope of a line when given an equation of the line, the graph of the line, or two points on the line~~. Slope will be described as rate of change and will be positive, negative, zero, or undefined~~ [Included in US and EKS]; ~~and~~
- b) write~~ing~~ the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
- c) graph linear equations in two variables. [Moved from SOL stem]

Functions

A.7 The student will investigate and analyze ~~function (~~linear and quadratic ~~)~~function families and their characteristics both algebraically and graphically, including
- a) determining whether a relation is a function;
- b) domain and range;
- c) zeros ~~of a function~~;
- d) ~~x- and y-~~intercepts;
- e) ~~finding the~~ values of a function for elements in its domain; and
- f) ~~making~~ connections between any two ~~and among multiple~~ representations of functions including concrete, verbal, numeric, graphic, and algebraic.

A.8 The student, given a data set or practical situation~~, in a real-world context~~, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

Statistics

~~A.9~~ ~~The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.~~[Included in AFDA.7 and Algebra II.11]

~~A.10~~ ~~The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.~~[Moved to 8.12]

A.9~~11~~ The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve ~~real-world~~practical problems, using mathematical models of linear and quadratic functions. ~~Mathematical models will include linear and quadratic functions.~~

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. All students are expected to achieve the Geometry standards. The course includes an emphasis on developing reasoning skills through the exploration of geometric relationships including~~, among other things,~~ properties of geometric figures, trigonometric relationships, and mathematical proofs~~reasoning to justify conclusions~~. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods.~~Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, algebraic methods, and verbal arguments. A gradual development of formal proof will be encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.~~

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these standards. Graphing utilities (calculators, computers, and other technology tools)~~Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators),~~ and dynamic geometry ~~software~~applications~~, and other appropriate technology tools~~ will be used to assist in teaching and learning. ~~Any technology that will enhance student learning should be used.~~

Reasoning, Lines, and Transformations

G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form; and
c) determining the validity of a logical argument. [Added from EKS] ~~using Venn diagrams torepresent set relationships~~ [Moved to DM.12]~~; and~~
~~d) using deductive reasoning.~~

G.2 The student will use the relationships between angles formed by two lines ~~cut~~ intersected by a transversal to

a) ~~determine whether~~prove two or more lines are parallel; and
b) ~~verify the parallelism, using algebraic and coordinate methods as well as deductive proofs;and~~
~~c)~~ solve ~~real-world~~ problems, including practical problems involving angles formed when parallel lines are ~~cut~~ intersected by a transversal.

G.3 The student will ~~use pictorial representations, including computer software, constructions, and coordinate methods, to~~ [Moved to US] solve problems involving symmetry and transformation. This will include

a) investigating and using formulas for ~~find~~determining distance, midpoint, and slope;
b) applying slope to verify and determine whether lines are parallel or perpendicular;
c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

G.4 The student will construct and justify the constructions of
a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle,
f) an angle congruent to a given angle; ~~and~~
g) a line parallel to a given line through a point not on the ~~given~~ line; and
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [Moved from EKS]

Triangles

G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include
a) ordering the sides by length, given ~~the~~ angle measures;
b) ordering the angles by degree measure, given ~~the~~ side lengths;
c) determinin~~g~~e whether a triangle exists; and
d) determinin~~g~~e the range in which the length of the third side must lie.
~~These concepts will be considered in the context of real-world situations.~~

G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent~~, using algebraic and coordinate methods as well as deductive proofs~~.

G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar~~, using algebraic and coordinate methods as well as deductive proofs~~.

G.8 The student will solve ~~real-world~~ problems, including practical problems, involving right triangles. This will include applying ~~by using the~~
a) Pythagorean Theorem and its converse;~~,~~
b) properties of special right triangles;~~,~~ and
c) ~~right triangle trigonometry~~trigonometric ratios.

Polygons and Circles

G.9 The student will verify ~~characteristics of quadrilaterals~~ and use properties of quadrilaterals to solve ~~real-world~~ problems, including practical problems.

G.10 The student will solve ~~real-world~~ problems, including practical problems, involving angles of convex polygons. This will include
a) determining the sum of the interior and/or exterior angles;
b) determining the measure of an interior and/or exterior angle; and
c) determining the number of sides of a regular polygon.

G.11 The student will solve problems, including practical problems, by applying properties of circles.~~use angles, arcs, chords, tangents, and secants to~~ This will include determining
a) ~~investigate, verify, and apply properties of circles~~[Moved to stem]angle measures formed by intersecting chords, secants, and/or tangents;
b) ~~solve real-world problems involving properties of circles~~[Moved to stem]lengths of segments formed by intersecting chords, secants, and/or tangents; ~~and~~
c) ~~find~~ arc length~~s~~; and
d) and area~~s~~ of a sector~~s in circles~~.

G.12 The student will solve problems involving ~~, given the coordinates of the center of a circle and a point on the circle, will write the~~ equations of ~~the~~ circles.

Three-Dimensional Figures

G.13 The student will use ~~formulas for~~ surface area and volume of three-dimensional objects to solve ~~real-world~~practical problems.

G.14 The student will ~~use~~apply the concepts of similarity ~~geometric objects in~~to two- or three-dimensional~~s~~ geometric figures. This will include ~~to~~

a) comparing~~e~~ ratios between ~~side~~ lengths, perimeters, areas, and volumes of similar figures;
b) determining~~e~~ how changes in one or more dimensions of a~~n~~ ~~object~~ figure affect area and/or volume of the ~~object~~figure;
c) determining~~e~~ how changes in area and/or volume of a~~n~~ ~~object~~ figure affect one or more dimensions of the ~~object~~figure; and
d) solving~~e~~ ~~real-world~~ problems, including practical problems, about similar geometric ~~objects~~figures.

Algebra, Functions, and Data Analysis

~~The following standards outline the content for a one-year course in Algebra, Functions, and Data Analysis.~~ This course is designed for students who have successfully completed the standards for Algebra I and may benefit from additional support in their transition to Algebra II. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated by practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or logarithmic equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from ~~real life~~relevant, applied contexts and situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication, both orally and in writing, throughout the course.

These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

~~The infusion of technology (graphing calculator and/or computer software) in this course will assist in modeling and investigating functions and data analysis.~~

Algebra and Functions

AFDA.1 The student will investigate and analyze ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function families and their characteristics. Key concepts include

a) domain and range~~continuity~~; [Reordered]
b) intervals on which a function is increasing or decreasing~~local and absolute maxima and minima~~ [Reordered];
c) ~~domain and range~~maxima and minima [Reordered];
d) zeros;
e) intercepts;
f) values of a function for elements in its domain~~intervals in which the function is increasing/decreasing~~ [Reordered];
g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic;
h) end behaviors; and
~~h~~i) vertical and horizontal asymptotes.

AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a ~~function (~~linear, quadratic, exponential, and logarithmic~~)~~ function.

AFDA.3 The student will collect and analyze data~~, and generate an~~determine the equation ~~for~~ of the curve of best fit in order to make predictions, and solve ~~(linear, quadratic, exponential, and logarithmic) of best fit to model real-world~~practical problems ~~or applications~~ using models of linear, quadratic, and exponential functions. ~~Students will use the best-fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphicalmodels.~~

AFDA.4 The student will ~~transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and~~ use ~~appropriate~~ multiple representations of functions for analysis, interpretation, and prediction.

AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Data Analysis

AFDA.6 The student will calculate probabilities. Key concepts include
a) conditional probability;
b) dependent and independent events;
c) ~~addition and multiplication rules~~mutually exclusive events;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

AFDA.7 The student will ~~analyze the normal distribution. Key concepts include:~~
a) describe the distribution of a univariate data set using descriptive statistics~~characteristics of normally distributed data~~;
b) identify and describe properties of a normal distribution~~percentiles~~;
c) interpret and compare *z*-scores for normally distributed data~~normalizing data using z-scores~~; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve~~area under the standard normal curve and probability~~.

AFDA.8 The student will design and conduct an experiment/survey. Key concepts include
a) sample size;
b) sampling technique;
c) controlling sources of bias and experimental error;
d) data collection; and
e) data analysis and reporting.

Algebra II

~~The standards below outline the content for a one-year course in Algebra II.~~ Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. ~~All students preparing for postsecondary and advanced technical studies are expected to achieve the Algebra II standards.~~ A thorough treatment of advanced algebraic concepts will be provided through the study of functions, ~~"families of functions,"~~ equations, inequalities, systems of equations and inequalities, polynomials, rational and radical equations, complex numbers, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results should also permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of ~~graphs~~functions" from a given ~~graph~~ "parent" function and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

~~Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving and verifying solutions to equations and inequalities. Any other available technology that will enhance student learning should be used.~~

Expressions and Operations

AII.1 The student, given rational, radical, or polynomial expressions, will

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;

b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;

~~c)~~ ~~write radical expressions as expressions containing rational exponents and vice versa~~[Moved to EKS]~~;~~ and

~~d~~c) factor polynomials completely in one or two variables.

~~AII. 2*~~ ~~The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.~~
~~*Standard AII.2 will be assessed in the Functions and Statistics reporting category. (Revised March 2011)~~[Moved to AII.5 in the Functions strand]

AII. 2~~3~~ The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*~~, and identify field properties that are valid for the complex numbers~~.

Equations and Inequalities

AII. 3~~4~~ The student will solve, algebraically and graphically,
a) absolute value linear equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

~~Graphing calculators will be used for solving and for confirming the algebraic solutions.~~ [Moved to EKS]

AII. 4~~5~~ The student will solve ~~nonlinear~~ systems of ~~equations, including~~ linear-quadratic and quadratic-quadratic equations, algebraically and graphically. ~~Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.~~ [Moved to EKS]

Functions

AII. 5~~2~~ The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve ~~real-world~~practical problems, including writing the first n terms, ~~finding~~ determining the n^{th} term, and evaluating summation formulas. Notation will include $\sum$ and a_n.[Moved from Expressions and Operations strand]

AII.6 For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, ~~T~~the student will
a) recognize the general shape of function ~~(absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic)~~ families; and ~~will~~
b) use knowledge of transformations to convert between graphic and symbolic forms of functions.

~~A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.~~ [Moved to EKS]

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions~~s~~ families algebraically and graphically. Key concepts include
a) domain, ~~and~~ range, and continuity~~including limited and discontinuous domains and ranges~~;
b) intervals in which a function is increasing or decreasing; [Reordered]
c) maxima and minima;
d) zeros;
~~c~~e) ~~*x*- and *y*-~~intercepts;
~~d~~f) ~~intervals in which a function is increasing or decreasing~~values of a function for elements in its domain;
~~e~~g) connections between any two representations of functions including concrete, verbal, numeric, graphic, and algebraic~~asymptotes~~;
~~f~~h) end behavior;
i) vertical and horizontal asymptotes; [Reordered]
~~g~~j) inverse of a function; and
~~h~~k) composition of ~~multiple~~ functions algebraically.

~~Graphing calculators will be used as a tool to assist in investigation of functions.~~ [Moved to EKS]

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Statistics

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit~~,~~ in order to make predictions, and solve ~~real-world~~practical problems, using mathematical models of linear, quadratic, and exponential functions. ~~Mathematical models will include polynomial, exponential, and logarithmic functions.~~

AII.10 The student will ~~identify~~represent~~, create,~~ and solve ~~real-world~~problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII.11 The student will

a) describe the distribution of a univariate data set using descriptive statistics;
b) identify and describe properties of a normal distribution
c) interpret and compare z-scores for normally distributed data; and
d) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

AII.12 The student will compute and distinguish between permutations and combinations ~~and use technology for applications~~.

Trigonometry

~~The standards below outline the content for a one-semester course in trigonometry. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards.~~ A thorough treatment of trigonometry will be provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should also be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

~~Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving and verifying solutions to trigonometric equations and inequalities.~~

T.1 The student, given a point ~~other than the origin~~ on the terminal side of an~~the~~ angle in standard position, or the value of the trigonometric function of the angle, will ~~use~~determine ~~the definitions of the six trigonometric functions to find~~ the sine, cosine, tangent, cotangent, secant, and cosecant of the angle ~~in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.~~

T.2 ~~The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.~~[Combined into T.1]
The student will develop and apply the properties of the unit circle in degrees and radians. [Moved and rewritten from T.3]

T.9~~3~~ ~~The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.~~[Moved to T.2]
The student will solve problems, including practical problems, involving
a) arc length and area of sectors in circles using radians and degrees; and
b) linear and angular velocity.

T.7~~4~~ The student will ~~find~~determine~~, with the aid of a calculator,~~ the value of any trigonometric function and inverse trigonometric function.

T.5 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

T.3~~6~~ The student, given one of the six trigonometric functions in standard form, will
a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a two-period interval; and
d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

T.4~~7~~ The student will graph the six inverse trigonometric functions.~~identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.~~ [Included in EKS]

T.6~~8~~ The student will solve trigonometric equations and inequalities~~that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities~~.

T.8~~9~~ The student will ~~identify, create,~~ and solve ~~real-world~~practical problems involving triangles. ~~Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.~~ [Included in EKS]

~~Algebra II and Trigonometry~~

~~The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.~~

~~Expressions and Operations~~

~~AII/T.1 The student, given rational, radical, or polynomial expressions, will~~
~~a) add, subtract, multiply, divide, and simplify rational algebraic expressions;~~
~~b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;~~
~~c) write radical expressions as expressions containing rational exponents and vice versa; and~~
~~d) factor polynomials completely.~~

~~AII/T.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.~~

~~AII/T.3 The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.~~

~~Equations and Inequalities~~

~~AII/T.4 The student will solve, algebraically and graphically,~~
~~a) absolute value equations and inequalities;~~
~~b) quadratic equations over the set of complex numbers;~~
~~c) equations containing rational algebraic expressions; and~~
~~d) equations containing radical expressions.~~
~~Graphing calculators will be used for solving and for confirming the algebraic solutions.~~

~~AII/T.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.~~

~~Functions~~

~~AII/T.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.~~

~~AII/T.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include~~
~~a) domain and range, including limited and discontinuous domains and ranges;~~
~~b) zeros;~~
~~c) *x*- and *y*-intercepts;~~
~~d) intervals in which a function is increasing or decreasing;~~
~~e) asymptotes;~~
~~f) end behavior;~~
~~g) inverse of a function; and~~
~~h) composition of multiple functions.~~
~~Graphing calculators will be used as a tool to assist in the investigation of functions.~~

~~AII/T.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.~~

~~Statistics~~

~~AII/T.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.~~

~~AII/T.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.~~

~~AII/T.11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.~~

~~AII/T.12 The student will compute and distinguish between permutations and combinations and use technology for applications.~~

~~Trigonometry~~

~~AII/T.13 The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.~~

~~AII/T.14 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.~~

~~AII/T.15 The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.~~

~~AII/T.16 The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.~~

~~AII/T.17 The student will verify basic trigonometric identities and make substitutions, using the basic identities.~~

~~AII/T.18 The student, given one of the six trigonometric functions in standard form, will~~
~~a) state the domain and the range of the function;~~
~~b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;~~
~~c) sketch the graph of the function by using transformations for at least a two-period interval; and~~
~~d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.~~

~~AII/T.19 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.~~

~~AII/T.20 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.~~

~~AII/T.21 The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.~~

Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry. ~~Students who successfully complete the standards for this course may earn credit toward meeting the mathematics graduation requirement. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student's course of study and treated as a review.~~

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving ~~per se~~ should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course~~ and may include programming a graphing calculator or scripting a problem solution in a database or spreadsheet~~. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

COM.1 The student will design and apply computer programs~~ming techniques and skills~~ to solve practical ~~real-world~~ problems in mathematics arising from ~~consumer,~~ business~~,~~ and ~~other~~ applications in mathematics. ~~Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.~~

~~*~~COM.2 The student will design, write, document, test, and debug, ~~and document~~ a computer program. ~~Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.~~[Moved to US]

~~*~~COM.3 The student will write program specifications that define the constraints of a given problem. ~~These specifications will include descriptions of preconditions, postconditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.~~[Moved to US]

~~*~~COM.4 The student will design a step-by-step ~~plan (~~algorithm~~)~~ to solve a given problem. ~~The plan will be in the form of a program flowchart, pseudo-code, hierarchy chart, and/or data-flow diagram.~~[Moved to US]

~~*~~COM.5 The student will divide a given problem into ~~manageable sections (~~modules~~)~~ by task and implement the solution. ~~The modules will include an appropriate user-defined functions, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.~~

~~*~~COM.10~~6~~ The student will design and implement the input phase of a program, which will include designing screen layout, ~~and~~ getting information into the program by way of user interaction~~, data statements,~~ and/or file input, and validating input. ~~The input phase will also include methods of filtering out invalid data (error trapping).~~

~~*~~COM.11~~7~~ The student will design and implement the output phase of a computer program, which will include designing output layout, accessing ~~a variety of~~ available output devices, using output statements, and labeling results.

COM.12~~8~~ The student will design and implement computer graphics to enhance output~~, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.~~

COM.15~~9~~ The student will define ~~simple~~and use appropriate variable data types that include integer, real (fixed and scientific notation), character, string, ~~and~~ Boolean, and object.

~~COM.10 The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.~~ [Combined with COM.15]

COM.16~~11~~ The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, ~~variables' addresses, pointers,~~ parameter passing, scope of variables, and local versus global variables.

~~*~~COM.6~~12~~ The student will translate ~~a~~ mathematical expressions into ~~a computer~~ programming ~~statement~~expressions~~, which involves~~ by declaring variables, writing assignment statements, and using the order of operations.

COM.7~~13~~ The student will select and ~~implement~~ call ~~built in (~~library~~)~~ functions ~~in~~to process~~ing~~ data, as appropriate.

COM. 8~~14~~ The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.

COM.13~~15~~ The student will implement various mechanisms for performing iteration with an algorithm~~loops, including iterative loops. Other topics will include single entry point, single exit point, preconditions, and postconditions.~~

COM.14~~16~~ The student will select and implement appropriate data structures, including arrays (one-- and/or two-dimensional ~~and/or multidimensional~~), ~~files,~~ and ~~records~~objects. ~~Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.~~

~~*~~COM.9~~17~~ The student will implement pre- ~~existing~~defined algorithms, including sort routines, search routines, and simple animation routines.

COM.17~~18~~ The student will test a program, using an appropriate set of data. The ~~set of~~ test data should include boundary cases and test all branches of a program. ~~be appropriate and complete for the type of program being tested.~~

COM.18~~19~~ The student will debug a program~~,~~ using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, other debugging tools available in the programming environment), and identify the difference ~~between~~ among syntax errors, runtime errors, and logic errors.

~~COM.20 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.~~ [Included in COM.2]

Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with a~~n asterisk~~ dagger (~~*~~†) would apply. ~~Students enrolled in this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra II.~~ The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

~~A graphing calculator is essential for every student taking the Probability and Statistics course and is required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a computer, however. In the absence of a computer for student use, teachers may provide students with examples of computer output generated by a statistical software package.~~

~~*~~PS.1† The student will analyze graphical displays of univariate data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers.~~Appropriate technology will be used to create graphical displays.~~[Moved to EKS]

~~*~~PS.2† The student will analyze numerical characteristics of univariate data sets to describe patterns and departures from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

~~*~~PS.3† The student will compare distributions of two or more univariate data sets, numerically and graphically, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

~~*~~PS.4† The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

PS.5 The student will ~~find~~determine and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.

PS.6 The student will make logarithmic and power transformations to achieve linearity.

PS.7† The student, using two-way tables and other graphical displays, will analyze categorical data to describe patterns and departure from patterns and to ~~find~~determine marginal frequency and relative frequencies, including conditional frequencies.

~~*~~PS.8† The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

~~*~~PS.9† The student will plan and conduct a survey. The plan will address sampling techniques ~~(e.g., simple random and stratified)~~[Moved to EKS] and methods to reduce bias.

PS.10† The student will plan and conduct a~~n~~ well-designed experiment. The plan will address control, randomization, replication, blinding, and measurement of experimental error.

~~*~~PS.11[†] The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

~~*~~PS.12 The student will ~~find~~determine probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

~~*~~PS.13 The student will develop, interpret, and apply the binomial and geometric probability distributions for discrete random variables, including computing the mean and standard deviation for the binomial and geometric variables.

PS.14 The student will simulate probability distributions, including binomial and geometric.

PS.15 The student will identify random variables as independent or dependent and ~~find~~ determine the mean and standard deviations for random variables and sums and differences of independent random variables.

~~*~~PS.16[‡] The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities~~, using a table or graphing calculator~~[Moved to EKS].

~~*~~PS.17 The student, given data from a large sample, will ~~find~~determine and interpret appropriate point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, ~~and~~ difference between two means (independent and paired), and slope of a least-squares regression line.

PS.18 The student will apply and interpret the logic of an appropriate hypothesis-testing procedure. Tests will include large sample tests for proportion, mean, difference between two proportions, ~~and~~ difference between two means (independent and paired); ~~and~~ Chi-squared tests for goodness of fit, homogeneity of proportions, and independence; and slope of a least-squares regression line.

PS.19 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

PS.20 The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures~~using tables or graphing calculators~~.

Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with a~~n asterisk~~dagger (~~*~~†) would apply. ~~Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II~~.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

~~Modern technology (graphing calculators and/or computers) will be an integral component of this course.~~

~~*~~DM.1† The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. ~~Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).~~[Included in US]

~~*~~DM.2† The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

~~*~~DM.3† The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. ~~Graph coloring and chromatic number will be used.~~[Moved to EKS]

~~*~~DM.4 The student will apply algorithms~~, such as Kruskal's, Prim's, or Dijkstra's,~~ [Included in EKS] relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

~~*~~DM.10~~5~~ The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

~~*~~DM.11~~6~~ The student will solve linear programming problems. ~~Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.~~[Included in EKS]

DM.5~~7~~† The student will analyze and describe the issue of fair division in discrete and continuous cases. ~~(e.g., cake cutting, estate division~~[Included in EKS]~~)~~. ~~Algorithms for continuous and discrete cases will be applied.~~

DM.6~~8~~† The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

DM.7~~9~~ The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

DM.12~~10~~ The student will use the recursive process and difference equations with the aid of appropriate technology to generate
a) compound interest;
b) sequences and series;
c) fractals;
d) population growth models; and
e) the Fibonacci sequence.

DM.8~~11~~ The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. ~~These will include~~
~~a)~~ ~~bubble sort, merge sort, and network sort; and~~
~~b)~~ ~~ISBN, UPC, Zip, and banking codes.~~[Included in EKS]

DM.9~~12~~† The student will select, justify, and apply an appropriate technique to solve a logic problem. ~~Techniques will include Venn diagrams, truth tables, and matrices.~~[Included in EKS]

DM.13 The student will apply the formulas of combinatorics in the areas of
a) the Fundamental (Basic) Counting Principle;
b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

Mathematical Analysis

~~The standards below outline the content for a one-year course in Mathematical Analysis.~~ Students enrolled in Mathematical Analysis are assumed to have mastered Geometry and Algebra II concepts~~ and have some exposure to trigonometry~~. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

~~Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying solutions to equations and inequalities.~~

MA.1 The student will investigate and identify the ~~characteristics~~ properties of polynomial, rational, piecewise, and step ~~and rational~~ functions and ~~use these to~~ sketch their graphs~~ of the functions~~. ~~This will include determining zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.~~[Included in EKS]

MA.3~~2~~ The student will apply compositions of functions and inverses of functions to ~~real-world~~practical situations~~.~~ ~~Analytical methods and graphing utilities will be used~~ [Included in EKS] ~~to~~ and investigate and verify the domain and range of resulting functions.

MA.5~~3~~ The student will investigate and describe the continuity of functions~~, using graphs and algebraic methods~~. [Moved to EKS]

MA. 12~~4~~ The student will expand binomials having positive integral exponents~~ through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle~~.[Included in EKS]

MA.13~~5~~ The student will ~~find~~determine the sum ~~(sigma notation included)~~ of finite and infinite convergent series~~, which will lead to an intuitive approach to a limit~~.

MA.14~~6~~ The student will use mathematical induction to prove formulas and mathematical statements.

MA.4~~7~~ The student will ~~find~~determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. ~~A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.~~[Included in EKS]

MA.6~~8~~ The student will investigate, graph and identify the ~~characteristics~~ properties of conic sections from equations in ~~(*h*, *k*)~~vertex and standard form~~s~~. ~~Transformations in the coordinate plane will be used to graph conic sections.~~ [Included in EKS]

MA.2~~9~~ The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, ~~in order to graph these functions and~~ solve equations, and solve ~~real-world~~practical problems. ~~This will include the role of e, natural and common logarithms,laws of exponents and logarithms, and the solution of logarithmic and exponential equations.~~ [Included in EKS]

MA.9~~10~~ The student will investigate and identify the characteristics of the graphs of polar equations.~~,~~ ~~using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.~~[Included in EKS]

MA.7~~11~~ The student will perform operations with vectors in the coordinate plane and solve ~~real-world~~ practical problems using vectors. ~~This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.~~[Included in EKS]

MA.10~~12~~ The student will use parametric equations to model and solve ~~application~~ practical problems.

MA.8~~13~~ The student will identify, create, and solve ~~real-world~~practical problems involving triangles. ~~Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.~~[Included in the EKS.]

MA.11~~14~~ The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.