Absolute Value Equations and Inequalities

Reporting Category Equations and Inequalities

Topic Solving absolute value equations and inequalities

Primary SOL AII.4a The student will solve, algebraically and graphically, absolute value equations and inequalities. Graphing calculators will be used for solving and for confirming the algebraic solutions.

Related SOL AII.6

Materials
- Graphing calculators
- Activity 1 Cards (attached)
- Three attached handouts

Vocabulary

union, intersection, linear inequality, absolute value, distance, linear equation (earlier grades)

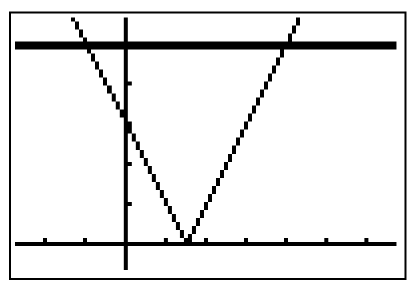
compound inequality, absolute value inequality (AII.4a)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Invoke students' prior learning by asking questions about absolute value. Use questions such as the following:
    - What is absolute value?
    - What does the graph of the absolute value of a linear function look like?
    - What are the solutions to $|x| = 5$?
    - What are some solutions to $|x| < 5$?
    - What are some solutions to $|x - 2| \geq 4$?

    Explain to students that the objective of the lesson is to learn three methods for solving absolute value equations and inequalities, which will give them greater insight into the solution process.

Method 1: Graphing

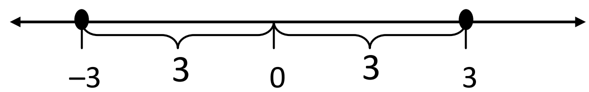


2. Ask students to graph $y = |2x - 3|$ and $y = 5$ by hand and then on their graphing calculators.
3. Next, ask students how they might use the graphs they generated to solve the following equation and inequalities:

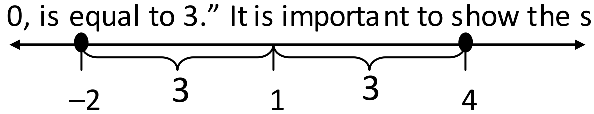
    a. $|2x-3| = 5$ b. $|2x-3| < 5$ c. $|2x-3| \geq 5$
4. Have students solve each one, using the graphs.
5. Distribute copies of the attached Solve by Graphing handout, and have students complete it, using the graphing method.

Method 2: Absolute Value as Distance

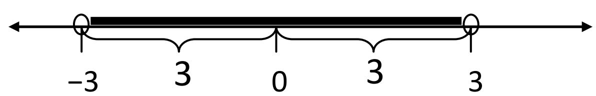
6. Discuss the statement, "Absolute value represents distance." Ask students whether this is true, and if so, what it means. Have the class discuss $|-5| = 5$ in terms of distance.
7. Ask what values make $|x| = 3$ true, and discuss this in terms of distance. Summarize responses by saying, "We can describe the solution set as the set of points whose distance from the origin is equal to 3." Show the solution set on a number line.



8. Ask what changes in the solution set when we want to solve $|x-1| = 3$. Summarize responses by saying, "We can describe the solution set as the set of points whose distance from 1, rather than 0, is equal to 3." It is important to show the solution set on a number line.



9. Ask, "If the solution to $|x| = 3$ is the set of points whose distance from the origin is *equal* to 3, then what is the solution set to $|x| < 3$?" Discuss this question, asking students to identify some values of *x* that make $|x| < 3$ true. They should come to see that the solution set is the set of points whose distance from the origin is *less than* 3. Show the solution set on a number line.



10. Ask how the graph of this solution set would be different if you were solving $|x| \le 3$. If you were solving $|x| > 3$. If you were solving $|x| \le 3$.
11. Instruct students to use the distance method to solve the following examples:

a. $|x-2| \le 3$ b. $|x+5| > 7$ c. $|x-4| \ge \frac{3}{4}$ d. $|x-a| < b$

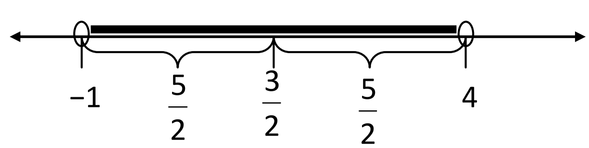
Then, have them state the solution of each example, using the sentence frame in the box below. Finally, have them graph each solution set on a number line.

Sentence Frame for Solution Set
" The solution set to <u>*given equation or inequality*</u>, is the set of points whose distance from ________________ is <u>*equal to, less than, etc.*</u> ________."

12. Discuss the validity of $|ab| = |a| \cdot |b|$. Ask whether it is true. Ask whether students can think of an example in which it is not true. Once its validity is established, show this example:

$$|2x-3| > 5 \Rightarrow \left|2\left(x-\frac{3}{2}\right)\right| > 5 \Rightarrow |2|\left|x-\frac{3}{2}\right| > 5 \Rightarrow 2\left|x-\frac{3}{2}\right| > 5 \Rightarrow \left|x-\frac{3}{2}\right| > \frac{5}{2}$$

Show on a number line that the solution set is the set of points whose distance from $\frac{3}{2}$ is less than $\frac{5}{2}$.

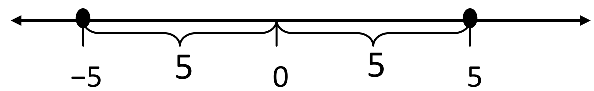


Therefore, we can write the solution set in set-builder notation as $\{x \mid -1 < x < 4\}$.

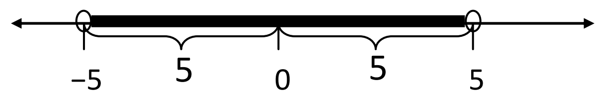
Method 3: Absolute Value as Compound Inequality

13. Explain the method of writing absolute value equations and inequalities as compound statements. Students should make the connection quickly if this is the third method discussed.

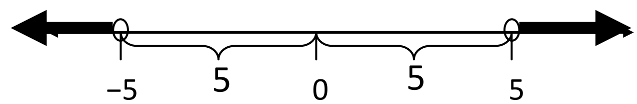
    $|x| = 5 \Rightarrow x = 5$ OR $x = -5$

    $|x| < 5 \Rightarrow x > -5$ AND $x < 5$

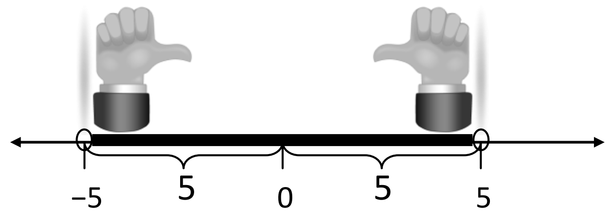
    $|x| > 5 \Rightarrow x < -5$ OR $x > 5$

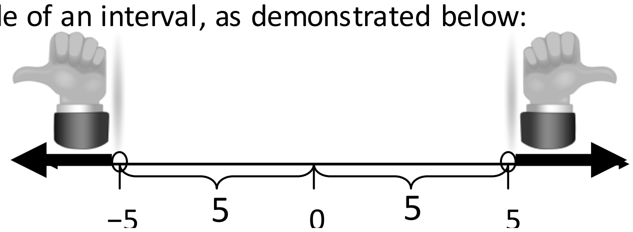
14. Have students write the following as compound statements, solve each branch, and graph the solution set to each: $|x + 2| = 6$ $\quad |2x - 3| \le 7$ $\quad |3x - 5| \ge 4$
15. Give every pair of students a set of Activity 1 Cards. Explain that student pairs will play a game to match an inequality with the graph of its solution set, the statement describing its solution set, and the corresponding compound statement. Have pairs shuffle their cards and deal 8 cards each. In turn, each player places one card on the table. If there is a corresponding card already on the table, the player places his/her card on top of that card to create a stack; if not, the player starts a new stack. At the end of the game, six stacks should have been created. When a student places the fourth card on any stack, that student collects that stack. The player with the most stacks wins. Once the game becomes too repetitive with these cards, have students create their own game cards in the same manner by writing inequalities, accompanying statements describing the solution sets, accompanying graphs, and accompanying compound inequalities.
16. Distribute copies of the attached Activity 2 Problems, and have students work in pairs to come up with three equations or inequalities that the provided graphs can help them solve. Have them also write the solutions.
17. The Activity 3 Problems are designed as a review and practice session on these topics, with students assisting each other through the review process. Set up six workstations with each station containing multiple sheets showing one set of problems. Be sure that each station has enough sheets for every student to have one. Place a poster at each station with the answers and full solutions to the problems. Put students into groups at the six stations. Give each group 7 to 10 minutes per station to complete the problems found there, working together and checking answers. Set a timer so that the groups know when to stop working and rotate to the next station.

Assessment

- **Questions**
  - You have noticed when you graph certain absolute value inequalities that the solution set is everything within an interval between two points, as demonstrated below:



    In other absolute value inequalities, the solution set is everything on the number line outside of an interval, as demonstrated below:



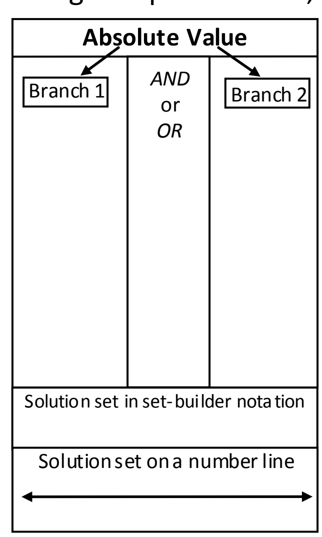
    What types of absolute value inequalities lead to each of these kinds of solution sets? Why?
  - Given any interval between two given numbers, how can you write an absolute value inequality to produce that interval as its solution set?
- **Journal/Writing Prompts**
  - Discuss the three ways of solving absolute value equations and inequalities, including which one you think makes the most sense and why.
  - Explain the steps you would use to solve an absolute value inequality, using your method of choice.
  - Write a fictitious story about the history of absolute value.

Extensions and Connections (for all students)

- Have students solve problems where the argument of the absolute value is quadratic.
- Have students work in groups to reteach one of the methods, and record a video for student review.
- Have students develop their own mnemonic to remember the connecting compound word, *and* or *or*, for each absolute value equation or inequality type.
- Have students solve $2 \leq |x| \leq 5$, using a distance argument.

Strategies for Differentiation

- When teaching the compound statement method, consider using a template like the one shown at right.
- Provide additional sentence frames to help students articulate solution sets to absolute value inequalities.
- Restrict the number of game cards to two representations for students who struggle with multiple representations.



- Construct a number line on the floor, and have students physically represent distances from zero and solution sets to absolute value equations and inequalities.
- Allow students to use talking graphing calculators as they interpret graphs and points of intersection.

Solve by Graphing

Graph each of the following on a number line, and then solve:

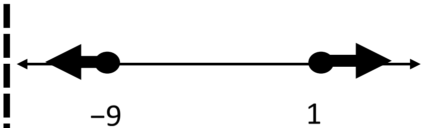
1. $3 < x \le 5$
2. $x < -1$ or $x \ge 5$
3. $x \ge 0$ and $x < 4$
4. $x \ge 4$ and $x < 7$
5. $x < -5$ and $x > 1$
6. $x \ge 4$ and $x > 7$
7. $x \ge 4$ or $x > 7$
8. $x > -1$ or $x < 3$

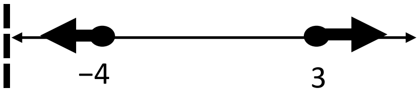
Graph on a number line the real numbers that satisfy each of the following:

9. $|x| = 5$
10. $|x| = 0$
11. $|x| = \le 8$
12. $|x| \ge 6$
13. $|x| \ge -1$
14. $|x| = \le 0$
15. $|x - 3| \le 5$
16. $|x + 3| \le 6$
17. $|x - 4| \ge 3$
18. $|2x - 5| \le 16$

Activity 1 Cards

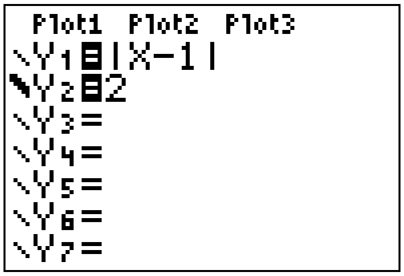
Copy cards on cardstock, and cut apart on the dotted lines.

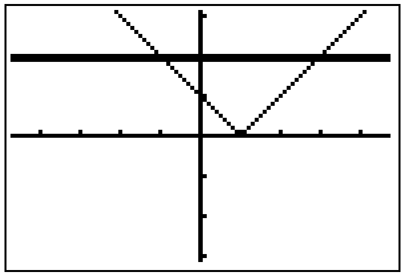
$\lvert x+5\rvert \leq 4$	The set of points whose distance from −5 is less than or equal to 4	
$x+5 \geq -4$ *and* $x+5 \leq 4$	The set of points whose distance from 5 is less than 4	
$\lvert x-5\rvert < 4$	$x-5 \geq -4$ *and* $x-5 \leq 4$	
$\lvert x+4\rvert \geq 5$	The set of points whose distance from −4 is greater than or equal to 5	$x+4 \leq -5$ *or* $x+4 \geq 5$

$\lvert 2x - 1 \rvert \le 7$	The set of points whose distance from $\frac{1}{2}$ is less than or equal to $\frac{7}{2}$	
$2x - 1 \ge -7$ *and* $2x - 1 \le 7$	$\lvert 2x + 1 \rvert \ge 7$	The set of points whose distance from $-\frac{1}{2}$ is greater than or equal to $\frac{7}{2}$
	$2x + 1 \le -7$ *or* $2x + 1 \ge 7$	$\lvert x - 2 \rvert \le 7$
The set of points whose distance from 2 is less than or equal to 7		$x - 2 \ge -7$ *and* $x - 2 \le 7$

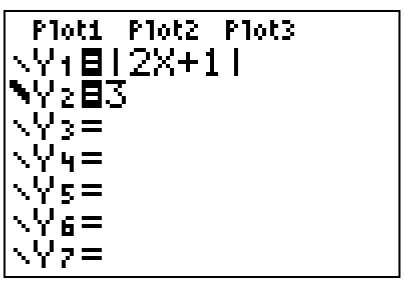
Activity 2 Problems

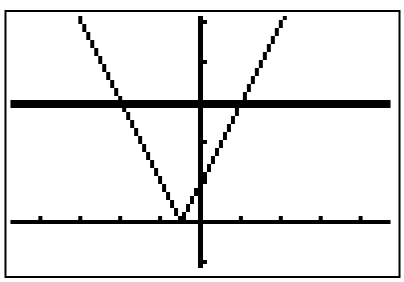
For each problem, write an absolute value equation and two inequalities that can be solved using the given graph. Then, solve, write the solution set in set-builder notation, and graph the solution set on a number line.

1. 



Absolute value equation	**Inequality 1**	**Inequality 2**
Solution set in set-builder notation	**Solution set in set-builder notation**	**Solution set in set-builder notation**
Graph of solution set	**Graph of solution set**	**Graph of solution set**

2. 



Absolute value equation	**Inequality 1**	**Inequality 2**
Solution set in set-builder notation	**Solution set in set-builder notation**	**Solution set in set-builder notation**
Graph of solution set	**Graph of solution set**	**Graph of solution set**

Activity 3 Problems

Station 1 Problems

Solve and graph the following:

1. $3(x - 2) \geq 5$
2. $6 - 4x \leq 10$
3. $\frac{x}{3} + 5 \leq 7$
4. $-5 \leq 2x + 1 \leq 3$
5. $7x - (3 - 2x) \geq x - 3$

Station 2 Problems

Solve and graph the following:

1. $3x + 6 \geq 4$ or $6x - 2 \geq 8$
2. $4x + 2 \geq 10$ or $-4x - 2 \geq 10$
3. $\frac{x}{-4} + 6 > 10$ and $\frac{2x - 1}{3} > 2$
4. $-5x < 10$ or $4x < 16$
5. Graph: $x > 5$ and $x > 9$

Station 3 Problems

Solve and graph the following:

1. $|2x + 5| = 3$
2. $|5 - x| + 4 = 10$
3. $-2|x + 6| = -10$
4. $|2x - 3| \leq 5$
5. $|11 - 3x| + 6 > 10$

Station 4 Problems

Solve and graph the following:

1. $3|2x - 2| + 8 = 23$
2. $|3x - 5| \geq 4$
3. $4|-x + 4| < 12$
4. $|5 - (2x + 3)| \geq 4$
5. $|2 - x| < 9$

Station 5 Problems

Write an inequality for each of the following:

1. You have \$20,000 available to invest in stocks A and B. Write an inequality stating the restriction on A if at least \$3,000 must be invested in each stock.
2. The largest egg of any bird is that of the ostrich. An ostrich egg can reach 8 inches in length. The smallest egg is that of the hummingbird. Its eggs are approximately 0.4 inches in length. Write an inequality that represents the range of lengths of bird eggs.
3. On Pennsylvania's interstate highway, the speed limit is 55 mph. The minimum speed limit is 45 mph. Write a compound inequality that represents the allowable speeds.
4. You have \$60 to spend and a coupon that allows you to take \$10 off any purchase of \$50 or more at the department store. Write an inequality that describes the possible retail value of the items you can buy if you use this coupon. Write an inequality that describes the different amounts of money you can spend if you use the coupon.
5. Develop a context for the inequality $0 < 2x - 5 < 30$.
6. Develop a context for the inequality $|x - 7| > 12$.

Station 6 Problems

Set up and solve each of the following inequalities:

1. Maria has grades of 79, 67, 83, and 90 on four tests. What is the lowest grade she can make on the fifth test in order to have an average of at least 82?
2. Most fish can adjust to a change in water temperature of up to 15°F if the change is not too sudden. Suppose lake trout live comfortably in water that is 58°F. Write an inequality that represents the range of temperatures at which the lake trout can survive. Write an absolute value inequality that represents the same information.